Vectors Review Paper 2 KEY

1a. *[8 marks]*

(i) evidence of approach ***M1***

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}=\overrightarrow{\mathrm{AB}}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}-4\\6\\-1\end{pmatrix}$ ***AG N0***

(ii) for choosing **correct** vectors, ($\overrightarrow{\mathrm{AO}}$ with $\overrightarrow{\mathrm{AB}}$ or $\overrightarrow{\mathrm{OA}}$ with $\overrightarrow{\mathrm{BA}}$) ***(A1)(A1)***

Note: Using $\overrightarrow{\mathrm{AO}}$ with $\overrightarrow{\mathrm{BA}}$ will lead to $\pi-0.799$. If they then say $\mathrm{B\hat{A}O}=0.799$, this is a correct solution.

calculating $\overrightarrow{\mathrm{AO}}\bullet\overrightarrow{\mathrm{AB}}$, $\left|\overrightarrow{\mathrm{AO}}\right|$, $\left|\overrightarrow{\mathrm{AB}}\right|$ ***(A1)(A1)(A1)***

e.g. $d_1\bullet d_2=(-1)(-4)+(2)(6)+(-3)(-1)(=19)$

$|d_1|=\sqrt{(-1)^2+2^2+(-3)^2}(=\sqrt{14})$, $|d_2|=\sqrt{(-4)^2+6^2+(-1)^2}(=\sqrt{53})$

evidence of using the formula to find the angle ***M1***

e.g. $\cos\theta=\frac{(-1)(-4)+(2)(6)+(-3)(-1)}{\sqrt{(-1)^2+2^2+(-3)^2}\sqrt{(-4)^2+6^2+(-1)^2}}$, $\frac{19}{\sqrt{14}\sqrt{53}}$, $0.69751\ldots$

$\mathrm{B\hat{A}O}=0.799$ radians (accept 45.8°) ***A1 N3***

[8 marks]

1b. *[2 marks]*

two correct answers ***A1A1***

e.g. $(1, -2, 3)$, $(-3, 4, 2)$, $(-7, 10, 1)$, $(-11, 16, 0)$ ***N2***

[2 marks]

1c. *[6 marks]*

(i) $\boldsymbol{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ ***A2 N2***

(ii) C on L_2, so $\begin{pmatrix} k \\ -k \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$ ***(M1)***

evidence of equating components ***(A1)***

e.g. $1 - 3t = k$, $-2 + 4t = -k$, $5 = 3 + 2t$

one correct value $t = 1$, $k = -2$ (seen anywhere) ***(A1)***

coordinates of C are $(-2, 2, 5)$ ***A1 N3***

[6 marks]

1d. *[2 marks]*

for setting up one (or more) correct equation using $\begin{pmatrix} -2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ ***(M1)***

e.g. $3 + p = -2$, $-8 - 2p = 2$, $-p = 5$

$p = -5$ ***A1 N2***

[2 marks]

2. *[6 marks]*

evidence of appropriate approach ***(M1)***

e.g. $\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$

two correct equations ***A1A1***

e.g. $2 + 5s = 9 - 3t$, $3 - 3s = 2 + 5t$, $-1 + 2s = 2 - t$

attempting to solve the equations ***(M1)*** one correct parameter $s = 2$, $t = -1$ ***A1***

P is $(12, -3, 3)$ (accept $\begin{pmatrix} 12 \\ -3 \\ 3 \end{pmatrix}$) ***A1 N3***

[6 marks]

3. *[7 marks]*

correct substitutions for $\boldsymbol{v} \bullet \boldsymbol{w}$; $|\boldsymbol{v}|$; $|\boldsymbol{w}|$ ***(A1)(A1)(A1)***

e.g. $2k + (-3) \times (-2) + 6 \times 4$, $2k + 30$; $\sqrt{2^2 + (-3)^2 + 6^2}$, $\sqrt{49}$; $\sqrt{k^2 + (-2)^2 + 4^2}$, $\sqrt{k^2 + 20}$

evidence of substituting into the formula for scalar product ***(M1)***

e.g. $\frac{2k+30}{7 \times \sqrt{k^2+20}}$

correct substitution ***A1***

e.g. $\cos \frac{\pi}{3} = \frac{2k+30}{7 \times \sqrt{k^2+20}}$

$k = 18.8$ ***A2 N5***

[7 marks]

4a. *[4 marks]*

(i) $(3, -4, 0)$ ***A1 N1***

(ii) choosing velocity vector $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ ***(M1)***

finding magnitude of velocity vector ***(A1)***

e.g. $\sqrt{(-2)^2 + 3^2 + 1^2}$, $\sqrt{4 + 9 + 1}$

speed $= 3.74$ $\left(\sqrt{14}\right)$ ***A1 N2***

[4 marks]

4b. *[5 marks]*

(i) substituting $p=7$ ***(M1)***

$\mathrm{B}=(-11, 17, 7)$ ***A1 N2***

(ii) **METHOD 1**

appropriate method to find $\overrightarrow{\mathrm{AB}}$ or $\overrightarrow{\mathrm{BA}}$ ***(M1)***

e.g. $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\mathrm{A}-\mathrm{B}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}-14\\21\\7\end{pmatrix}$ or $\overrightarrow{\mathrm{BA}}=\begin{pmatrix}14\\-21\\-7\end{pmatrix}$ ***(A1)***

distance $=26.2$ $\left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 2

evidence of applying distance is speed × time ***(M2)***

e.g. 3.74×7

distance $=26.2$ $\left(7\sqrt{14}\right)$ ***A1 N3***

METHOD 3

attempt to find AB , AB ***(M1)***

e.g. $(3-(-11))^2+(-4-17)^2+(0-7))^2$, $\sqrt{(3-(-11))^2+(-4-17)^2+(0-7))^2}$

AB $=686$, AB $=\sqrt{686}$ ***(A1)***

distance AB $=26.2$ $\left(7\sqrt{14}\right)$ ***A1 N3***

[5 marks]

4c. *[7 marks]*

correct direction vectors $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$ ***(A1)(A1)***

$\begin{vmatrix} -1 \\ 2 \\ a \end{vmatrix} = \sqrt{a^2+5}$, $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix} = a + 8$ ***(A1)(A1)***

substituting ***M1***

e.g. $\cos 40^\circ = \frac{a+8}{\sqrt{14}\sqrt{a^2+5}}$

$a = 3.21$, $a = -0.990$ ***A1A1 N3***

[7 marks]

5a. *[2 marks]*

appropriate approach ***(M1)***

e.g. $\overrightarrow{\mathrm{AO}} + \overrightarrow{\mathrm{OB}}$, $B - A$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ***A1 N2***

[2 marks]

5b. *[2 marks]*

any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2 N2***

where $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

e.g. $\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} 2+t \\ -2-t \\ 5+t \end{pmatrix}$, $\boldsymbol{r} = 2\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k} + t(\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k})$

[2 marks]

5c. *[7 marks]*

choosing correct direction vectors $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ ***(A1)(A1)***

finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $= 1 \times 2 + -1 \times 1 + 1 \times 3$ $(= 4)$

magnitudes $\sqrt{1^2 + (-1)^2 + 1^2}$ $(= 1.73\ldots)$, $\sqrt{4+1+9}$ $(= 3.74\ldots)$

substitution into $\frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$ (accept $\theta = \frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$, but not $\sin\theta = \frac{\boldsymbol{u} \bullet \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$) ***M1***

e.g. $\cos\theta = \frac{1\times2+-1\times1+1\times3}{\sqrt{1^2+(-1)^2+1^2}\sqrt{2^2+1^2+3^2}}$, $\cos\theta = \frac{4}{\sqrt{42}}$

$\theta = 0.906$ (51.9°) ***A1 N5***

[7 marks]

5d. *[6 marks]*

METHOD 1 (from $\boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} t = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$

two **correct** equations ***A1A1***

e.g. $1 + t = 2 + 2s$, $-1 - t = 4 + s$, $4 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -3$, $s = -2$

C is $(-2, 2, 1)$ ***A1 N3***

METHOD 2 (from $\boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$

two **correct** equations ***A1A1***

e.g. $2 + t = 2 + 2s$, $-2 - t = 4 + s$, $5 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -4$, $s = -2$

C is $(-2, 2, 1)$ ***A1 N3***

[6 marks]

6. *[7 marks]*

appropriate approach ***(M1)***

eg $\begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $L_1 = L_2$

any two correct equations ***A1A1***

eg $10 + 2s = 2 + 3t$, $6 - 5s = 1 + 5t$, $-1 - 2s = -3 + 2t$

attempt to solve ***(M1)***

eg substituting one equation into another

one correct parameter ***A1***

eg $s = -1$, $t = 2$

correct substitution ***(A1)***

eg $2+3(2)$, $1+5(2)$, $-3+2(2)$

A $=(8, 11, 1)$ (accept column vector) ***A1 N4***

[7 marks]

7a. *[3 marks]*

(i) appropriate approach ***(M1)***

eg $\overrightarrow{\mathrm{AO}}+\overrightarrow{\mathrm{OB}}$, $\mathrm{B}-\mathrm{A}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}1\\3\\2\end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{AC}}=\begin{pmatrix}2\\4\\a\end{pmatrix}$ ***A1 N1***

[3 marks]

7b. *[4 marks]*

valid reasoning (seen anywhere) ***R1***

eg scalar product is zero, $\cos\frac{\pi}{2}=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

correct scalar product of **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ (may be seen in part (c)) ***(A1)***

eg $1(2)+3(4)+2(a)$

correct working for **their** $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(A1)***

eg $2a+14$, $2a=-14$

$a=-7$ ***A1 N3***

[4 marks]

7c. *[8 marks]*

correct magnitudes (may be seen in (b)) ***(A1)(A1)***

$\sqrt{1^2+3^2+2^2}\left(=\sqrt{14}\right)$, $\sqrt{2^2+4^2+a^2}\left(=\sqrt{20+a^2}\right)$

substitution into formula ***(M1)***

eg $\cos\theta\,\frac{1\times2+3\times4+2\times a}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+4^2+a^2}}$, $\frac{14+2a}{\sqrt{14}\sqrt{4+16+a^2}}$

simplification leading to required answer ***A1***

eg $\cos\theta=\frac{14+2a}{\sqrt{14}\sqrt{20+a^2}}$

$\cos\theta=\frac{2a+14}{\sqrt{14a^2+280}}$ ***AG N0***

[4 marks]

correct setup ***(A1)***

eg $\cos 1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos 1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

7d. *[4 marks]*

correct setup ***(A1)***

eg $\cos 1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve ***(M1)***

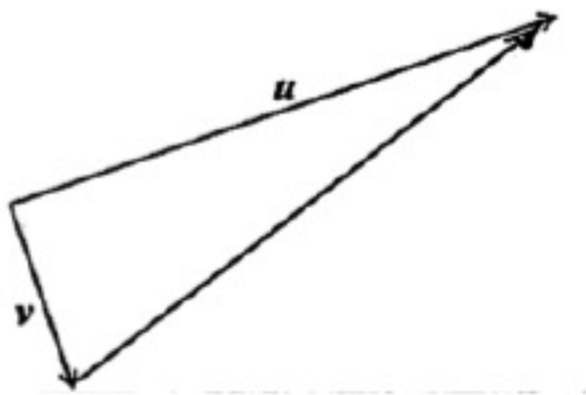
eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos 1.2=0$, attempt to square

$a=-3.25$ ***A2 N3***

[4 marks]

8a. *[2 marks]*

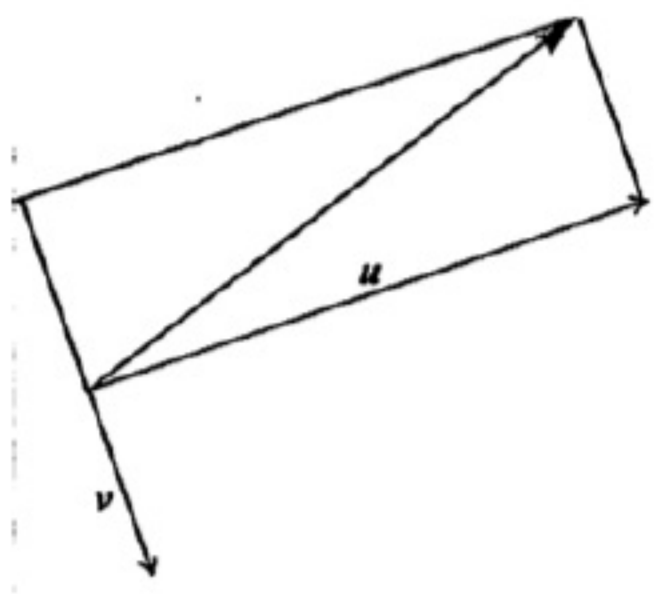
METHOD 1



A1A1 N2

Note: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

METHOD 2



A1A1 N2

Notes: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

Additional lines not required.

[2 marks]

8b. *[4 marks]*

Markscheme

evidence of setting scalar product equal to zero (seen anywhere) ***R1*** *eg* $\boldsymbol{u}\cdot\boldsymbol{v}=0,\ 15+2n+3=0$

correct expression for scalar product ***(A1)***

eg $3\times5+2\times n+1\times3,\ 2n+18=0$

attempt to solve equation ***(M1)***

eg $2n=-18$

$n=-9$ ***A1 N3***

[4 marks]

9a. *[6 marks]*

appropriate approach ***(M1)***

eg $\begin{pmatrix}11\\8\\2\end{pmatrix}+s\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\1\\-7\end{pmatrix}+t\begin{pmatrix}2\\1\\11\end{pmatrix}$, $L_1=L_2$

any **two** correct equations ***A1A1***

eg $11+4s=1+2t,\ 8+3s=1+t,\ 2-s=-7+11t$

attempt to solve system of equations ***(M1)***

eg $10+4s=2(7+3s),\begin{cases}4s-2t=-10\\3s-t=-7\end{cases}$

one correct parameter ***A1***

eg $s=-2,\ t=1$

$\mathrm{P}(3,2,4)$ (accept position vector) ***A1 N3***

[6 marks]

9b. *[5 marks]*

choosing correct direction vectors for L_1 and L_2 ***(A1)(A1)***

eg $\begin{pmatrix}4\\3\\-1\end{pmatrix},\begin{pmatrix}2\\1\\11\end{pmatrix}$ (or any scalar multiple) evidence of scalar product (with any vectors) ***(M1)***

eg $\boldsymbol{a}\cdot\boldsymbol{b}$, $\begin{pmatrix}4\\3\\-1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\11\end{pmatrix}$

correct substitution ***A1***

eg $4(2)+3(1)+(-1)(11),\ 8+3-11$

calculating $\boldsymbol{a}\cdot\boldsymbol{b}=0$ ***A1***

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular ***AG N0***

[5 marks]

9c. *[6 marks]*

Note: Candidates may take different approaches, which do not necessarily involve vectors.

In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\overrightarrow{\mathrm{QP}}$ or $\overrightarrow{\mathrm{PQ}}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\overrightarrow{\mathrm{QP}}=\begin{pmatrix}-4\\-3\\1\end{pmatrix}$, $\overrightarrow{\mathrm{PQ}}=\begin{pmatrix}7\\5\\3\end{pmatrix}-\begin{pmatrix}3\\2\\4\end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg $\overrightarrow{\mathrm{QP}}=\overrightarrow{\mathrm{PR}}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix}3\\2\\4\end{pmatrix}-\begin{pmatrix}7\\5\\3\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}-\begin{pmatrix}3\\2\\4\end{pmatrix},\begin{pmatrix}-4\\-3\\1\end{pmatrix}+\begin{pmatrix}3\\2\\4\end{pmatrix}$

$\mathrm{R}(-1,-1,5)$ (accept position vector) ***A1 N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***

eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***

eg P midpoint of QR, marked on diagram

valid approach to find **one** coordinate of mid-point ***(M1)***

eg $x_p=\frac{x_Q+x_R}{2}$, $2y_p=y_Q+y_R,\ \frac{1}{2}\left(z_Q+z_R\right)$

one correct substitution ***A1***

eg $x_R=3+(3-7),\ 2=\frac{5+y_R}{2}$, $4=\frac{1}{2}(z+3)$

correct working for one coordinate ***(A1)***

eg $x_R=3-4,\ 4-5=y_R,\ 8=(z+3)$

$\mathrm{R}(-1,-1,5)$ (accept position vector) ***A1 N3***

[6 marks]

10. *[6 marks]*

evidence of equating vectors ***(M1)***

e.g. $L_1=L_2$

for any **two** correct equations ***A1A1***

e.g. $2+s=3-t$, $5+2s=-3+3t$, $3+3s=8-4t$

attempting to solve the equations ***(M1)***

finding **one** correct parameter $(2=-1,t=2)$ ***A1*** the coordinates of T are $(1, 3, 0)$ ***A1 N3***

[6 marks]

11a. *[1 mark]*

L_1 ***A1 N1***

[1 mark]

11b. *[1 mark]*

$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ ***A1 N1***

[1 mark]

11c. *[5 marks]*

choosing correct direction vectors ***A1A1***

e.g. $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$

recognizing that $\boldsymbol{a} \bullet \boldsymbol{b} = 0$ ***M1***

correct substitution ***A1***

e.g. $-3 - 4 - a = 0$

$a = -7$ ***A1 N3***

[5 marks]