Vectors Review Paper 2

1a. *[8 marks]*

The point O has coordinates (0 , 0 , 0) , point A has coordinates (1 , – 2 , 3) and point B has coordinates (– 3 , 4 , 2) .

(i) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

(ii) Find $\mathrm{B\hat{A}O}$.

1b. *[2 marks]*

The line L_1 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} -4 \\ 6 \\ -1 \end{pmatrix}$.

Write down the coordinates of two points on L_1 .

1c. *[6 marks]*

The line L_2 passes through A and is parallel to $\overrightarrow{\mathrm{OB}}$.

(i) Find a vector equation for L_2 , giving your answer in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

(ii) Point $C(k, -k, 5)$ is on L_2 . Find the coordinates of C.

1d. *[2 marks]*

The line L_3 has equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -8 \\ 0 \end{pmatrix} + p \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ and passes through the point C.

Find the value of *p* at C.

2. *[6 marks]*

Two lines with equations $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} + s\begin{pmatrix} 5 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 9 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} -3 \\ 5 \\ -1 \end{pmatrix}$ intersect at the point P. Find the coordinates of P.

3. *[7 marks]*

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} k \\ -2 \\ 4 \end{pmatrix}$, for $k > 0$. The angle between ***v*** and ***w*** is $\frac{\pi}{3}$.

Find the value of k.

4a. *[4 marks]*

In this question, distance is in metres.

Toy airplanes fly in a straight line at a constant speed. Airplane 1 passes through a point A.

Its position, *p* seconds after it has passed through A, is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix} + p\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$.

(i) Write down the coordinates of A.

(ii) Find the speed of the airplane in ms^{-1}.

4b. *[5 marks]*

After seven seconds the airplane passes through a point B.

(i) Find the coordinates of B.

(ii) Find the distance the airplane has travelled during the seven seconds.

4c. *[7 marks]*

Airplane 2 passes through a point C. Its position *q* seconds after it passes through C is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix} + q\begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}, a \in \mathbb{R}.$$

The angle between the flight paths of Airplane 1 and Airplane 2 is 40°. Find the two values of *a*.

5a. *[2 marks]*

Line L_1 passes through points $\mathrm{A}(1, -1, 4)$ and $\mathrm{B}(2, -2, 5)$.

Find $\overrightarrow{\mathrm{AB}}$.

5b. *[2 marks]*

Find an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

5c. *[7 marks]*

Line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

Find the angle between L_1 and L_2.

5d. *[6 marks]*

The lines L_1 and L_2 intersect at point C. Find the coordinates of C.

6. *[7 marks]*

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

7a. *[3 marks]*

Consider the points A$(5, 2, 1)$, B$(6, 5, 3)$, and C$(7, 6, a + 1)$, $a \in \mathbb{R}$.

Find

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$.

7b. *[4 marks]*

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

Find the value of a for which $\theta = \frac{\pi}{2}$.

7c. *[8 marks]*

i. Show that $\cos q = \frac{2a+14}{\sqrt{14a^2+280}}$.

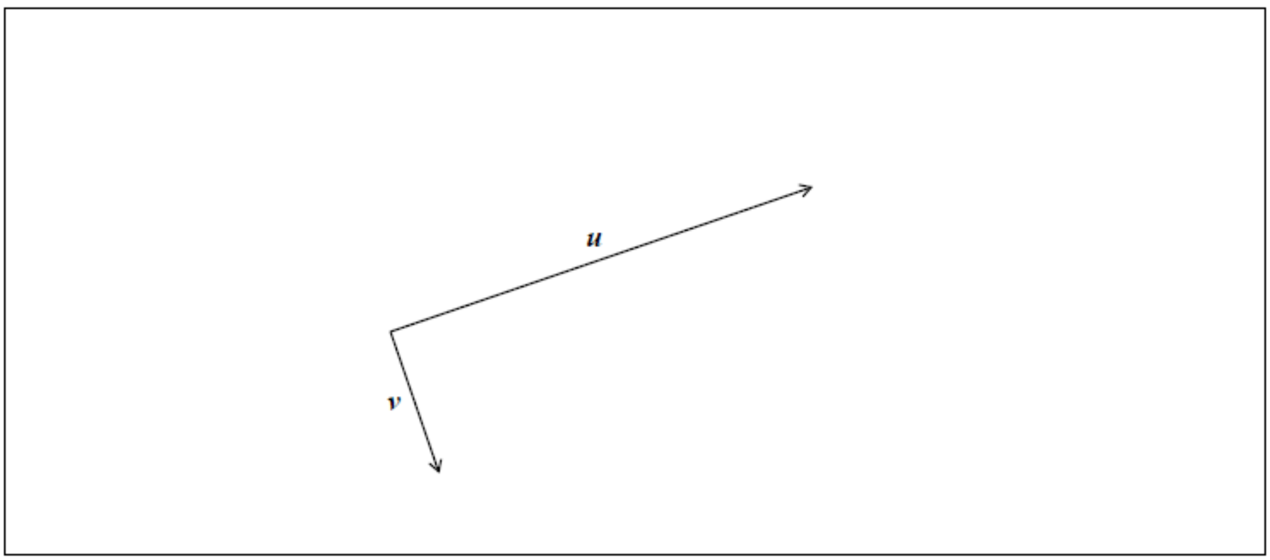
ii. Hence, find the value of a for which $\theta = 1.2$.

7d. *[4 marks]*

Hence, find the value of a for which $\theta = 1.2$.

8a. *[2 marks]*

The following diagram shows two perpendicular vectors ***u*** and ***v***.



Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above.

8b. *[4 marks]*

Given that $u = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find \(*n*\).

9a. *[6 marks]*

Consider the lines L_1 and L_2 with equations L_1 : $\boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and L_2 : $\boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

Find the coordinates of P.

9b. *[5 marks]*

Show that the lines are perpendicular.

9c. *[6 marks]*

The point $\mathrm{Q}(7, 5, 3)$ lies on L_1. The point R is the reflection of Q in the line L_2.

Find the coordinates of R.

10. *[6 marks]*

The line *L* is represented by $\boldsymbol{r}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix} + s\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and the line *L* by $\boldsymbol{r}_2 = \begin{pmatrix} 3 \\ -3 \\ 8 \end{pmatrix} + t\begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}$.

The lines *L* and *L* intersect at point T. Find the coordinates of T.

11a. *[1 mark]*

Consider the lines L_1 , L_2 , L_2 , and L_4 , with respective equations.

$$L_1: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

$$L_2: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + p \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$L_3: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + s \begin{pmatrix} -1 \\ 2 \\ -a \end{pmatrix}$$

$$L_4: \begin{pmatrix} x \\ y \\ z \end{pmatrix} = q \begin{pmatrix} -6 \\ 4 \\ -2 \end{pmatrix}$$

Write down the line that is parallel to L_4 .

11b. *[1 mark]*

Write down the position vector of the point of intersection of L_1 and L_2 .

11c. *[5 marks]*

Given that L_1 is perpendicular to L_3 , find the value of a .