Algebra Test Paper 2 KEY

1. (a) $\sum_{r=4}^{7} 2^r = 2^4 + 2^5 + 2^6 + 2^7$ (accept 16 + 32 + 64 + 128) A1 N1

(b) (i) **METHOD 1**

recognizing a GP (M1)

$u_1 = 2^4$, $r = 2$, $n = 27$ (A1)

correct substitution into formula for sum (A1)

e.g. $S_{27} = \frac{2^4(2^{27} - 1)}{2 - 1}$

$S_{27} = 2147483632$ A1 N4

METHOD 2

recognizing $\sum_{r=4}^{30} = \sum_{r=1}^{30} - \sum_{r=1}^{3}$ (M1)

recognizing GP with $u_1 = 2$, $r = 2$, $n = 30$ (A1)

correct substitution into formula for sum

$S_{30} = \frac{2(2^{30} - 1)}{2 - 1}$ (A1)

$= 2147483646$

$\sum_{r=4}^{30} 2^r = 2147483646 - (2 + 4 + 8)$

$= 2147483632$ A1 N4

(ii) valid reason (*e.g.* **infinite** GP, diverging series), **and** $r \geq 1$ (accept $r > 1$) R1R1 N2

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Algebra Test Paper 2 KEY

2. evidence of binomial expansion *(M1)*

eg selecting correct term, $\left(\frac{x}{a}\right)^6\left(\frac{a^2}{x}\right)^0+\binom{6}{1}\left(\frac{x}{a}\right)^5\left(\frac{a^2}{x}\right)^1+\dots$

evidence of identifying constant term in expansion for power 6 *(A1)*

eg $r=3$, 4 term

evidence of correct term (may be seen in equation) *A2*

eg $20\frac{a^6}{a^3}$, $\binom{6}{3}\left(\frac{x}{a}\right)^3\left(\frac{a^2}{x}\right)^3$

attempt to set up **their** equation *(M1)*

eg $\binom{6}{3}\left(\frac{x}{a}\right)^3\left(\frac{a^2}{x}\right)^3=1280$, $a^3=1280$

correct equation in one variable a *(A1)*

eg $20a^3=1280$, $a^3=64$

$a=4$ *A1 N4*

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3. (a) Plan A: 1000, 1080, 1160... Plan B: 1000, 1000(1.06), $1000(1.06)^2$…
2nd month: \$1060, 3rd month: \$1123.60 (A1)(A1) 2

(b) For Plan A, $T_{12}=a+11d$
$=1000+11(80)$ (M1)
$=\$1880$ (A1)

For Plan B, $T_{12}=1000(1.06)^{11}$ (M1)
$=\$1898$ (to the nearest dollar) (A1) 4

(c) (i) For Plan A, $S_{12}=\frac{12}{2}[2000+11(80)]$ (M1)
$=6(2880)$
$=\$17280$ (to the nearest dollar) (A1)

(ii) For Plan B, $S_{12}=\frac{1000(1.06^{12}-1)}{1.06-1}$ (M1)
$=\$16870$ (to the nearest dollar) (A1) 4

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Algebra Test Paper 2 KEY

4. ***Note:*** *Throughout this question, the first and last terms are interchangeable.*

(a) For recognizing the arithmetic sequence (M1)
$u_1 = 1, n = 20, u_{20} = 20$ ($u_1 = 1, n = 20, d = 1$) (A1)
Evidence of using sum of an AP M1

$S_{20} = \frac{(1+20)20}{2}$ (or $S = \frac{20}{2}(2\times1+19\times1)$) A1

$S_{20} = 210$ AG N0

(b) Let there be n cans in bottom row

Evidence of using $S_n = 3240$ (M1)

eg $\frac{(1+n)n}{2} = 3240, \frac{n}{2}(2+(n-1)) = 3240, \frac{n}{2}(2n+(n-1)(-1)) = 3240$

$n^2 + n - 6480 = 0$ A1

$n = 80$ or $n = -81$ (A1)

$n = 80$ A1 N2

(c) (i) Evidence of using $S = \frac{(1+n)n}{2}$ (M1)

$2S = n^2 + n$ A1

$n^2 + n - 2S = 0$ AG N0

(ii) **METHOD 1**

Substituting $S = 2100$

eg $n^2 + n - 4200 = 0$, $2100 = \frac{(1+n)n}{2}$ A1

EITHER

$n = 64.3, n = -65.3$ A1

Any valid reason which includes reference to integer being needed, R1

and pointing out that integer not possible here. R1 N1

eg n must be a (positive) integer, this equation does not have integer solutions.

OR

Discriminant = 16 801 A1

Valid reason which includes reference to integer being needed, R1

and pointing out that integer not possible here. R1 N1

eg this discriminant is not a perfect square, therefore no integer solution as needed.

METHOD 2
Trial and error
$S_{64} = 2080, S_{65} = 2145$ A1A1
Any valid reason which includes reference to integer being needed, R1
and pointing out that integer not possible here. R1 N1

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