

Series and Sequences, Binomial Theorem Review Paper 2

1. Use the binomial theorem to complete this expansion.

$$(3x + 2y)^4 = 81x^4 + 216x^3y + \dots$$

Working:

Answer:

.....

(Total 4 marks)

2. Determine the constant term in the expansion of $\left(x - \frac{2}{x^2}\right)^9$.

Working:

Answer:

.....

(Total 4 marks)

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3. Find the term containing x^{10} in the expansion of $(5 + 2x^2)^7$.

Working:

Answer:

.....

(Total 6 marks)

4. Find the coefficient of x^5 in the expansion of $(3x - 2)^8$.

Working:

Answer:

.....

(Total 4 marks)

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5. Find the coefficient of a^3b^4 in the expansion of $(5a + b)^7$.

Working:

Answer:

.....

(Total 4 marks)

6. Find the coefficient of a^5b^7 in the expansion of $(a + b)^{12}$.

Working:

Answer:

.....

(Total 4 marks)

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7. Consider the expansion of $\left(3x^2 - \frac{1}{x}\right)^9$.

- (a) How many terms are there in this expansion?
- (b) Find the constant term in this expansion.

Working:

Answers:

- (a)
- (b)

(Total 6 marks)

8. The first four terms of a sequence are 18, 54, 162, 486.

- (a) Use all four terms to show that this is a geometric sequence.

(2)

- (b) (i) Find an expression for the n^{th} term of this geometric sequence.

- (ii) If the n^{th} term of the sequence is 1062 882, find the value of n .

(4)

(Total 6 marks)

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9. \$1000 is invested at the beginning of each year for 10 years.

The rate of interest is fixed at 7.5% per annum. Interest is compounded annually.

Calculate, giving your answers to the nearest dollar

- (a) how much the first \$1000 is worth at the end of the ten years;
- (b) the total value of the investments at the end of the ten years.

Working:

Answers:

(a)

(b)

(Total 4 marks)

10. A sum of \$5000 is invested at a compound interest rate of 6.3% per annum.

- (a) Write down an expression for the value of the investment after n full years.
- (b) What will be the value of the investment at the end of five years?
- (c) The value of the investment will exceed \$10000 after n full years,
 - (i) Write down an inequality to represent this information.
 - (ii) Calculate the minimum value of n .

(Total 6 marks)

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11. In a geometric series, $u_1 = \frac{1}{81}$ and $u_4 = \frac{1}{3}$.

(a) Find the value of r .

(3)

(b) Find the smallest value of n for which $S_n > 40$.

(4)

(Total 7 marks)

12. Consider the arithmetic sequence 3, 9, 15, ..., 1353.

(a) Write down the common difference.

(1)

(b) Find the number of terms in the sequence.

(3)

(c) Find the sum of the sequence.

(2)

(Total 6 marks)

13. (a) Consider the geometric sequence $-3, 6, -12, 24, \dots$

(i) Write down the common ratio.

(ii) Find the 15th term.

Consider the sequence $x - 3, x + 1, 2x + 8, \dots$

(3)

(b) When $x = 5$, the sequence is geometric.

(i) Write down the first three terms.

(ii) Find the common ratio.

(2)

(c) Find the other value of x for which the sequence is geometric.

(4)

(d) For this value of x , find

(i) the common ratio;

(ii) the sum of the infinite sequence.

(3)

(Total 12 marks)

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14. An arithmetic sequence, u_1, u_2, u_3, \dots , has $d = 11$ and $u_{27} = 263$.

(a) Find u_1 .

(2)

(b) (i) Given that $u_n = 516$, find the value of n .

(ii) For this value of n , find S_n .

(4)

(Total 6 marks)

15. In an arithmetic sequence, $S_{40} = 1900$ and $u_{40} = 106$. Find the value of u_1 and of d .

(Total 6 marks)

16. Portable telephones are first sold in the country *Cellmania* in 1990. During 1990, the number of units sold is 160. In 1991, the number of units sold is 240 and in 1992, the number of units sold is 360.

In 1993 it was noticed that the annual sales formed a geometric sequence with first term 160, the 2nd and 3rd terms being 240 and 360 respectively.

(a) What is the common ratio of this sequence?

(1)

Assume that this trend in sales continues.

(b) How many units will be sold during 2002?

(3)

(c) In what year does the number of units sold first exceed 5000?

(4)

Between 1990 and 1992, the total number of units sold is 760.

(d) What is the total number of units sold between 1990 and 2002?

(2)

During this period, the total population of *Cellmania* remains approximately 80 000.

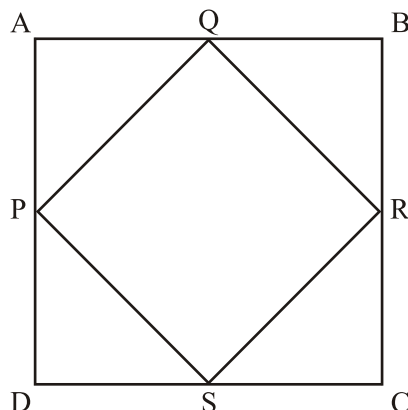
(e) Use this information to suggest a reason why the geometric growth in sales would not continue.

(1)

(Total 11 marks)

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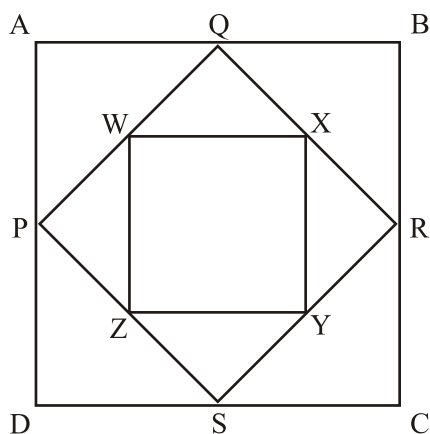
17. The diagram shows a square ABCD of side 4 cm. The midpoints P, Q, R, S of the sides are joined to form a **second** square.



- (a) (i) Show that $PQ = 2\sqrt{2}$ cm.
 (ii) Find the area of PQRS.

(3)

The midpoints W, X, Y, Z of the sides of PQRS are now joined to form a **third** square as shown.



- (b) (i) Write down the area of the **third** square, WXYZ.
 (ii) Show that the areas of ABCD, PQRS, and WXYZ form a geometric sequence. Find the common ratio of this sequence.

(3)

The process of forming smaller and smaller squares (by joining the midpoints) is **continued indefinitely**.

- (c) (i) Find the area of the 11th square.
 (ii) Calculate the sum of the areas of **all** the squares.

(4)

(Total 10 marks)

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18. Ashley and Billie are swimmers training for a competition.

(a) Ashley trains for 12 hours in the first week. She decides to increase the amount of time she spends training by 2 hours each week. Find the total number of hours she spends training during the first 15 weeks.

(3)

(b) Billie also trains for 12 hours in the first week. She decides to train for 10% longer each week than the previous week.

(i) Show that in the third week she trains for 14.52 hours.

(ii) Find the total number of hours she spends training during the first 15 weeks.

(4)

(c) In which week will the time Billie spends training first exceed 50 hours?

(4)

(Total 11 marks)

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19. The diagrams below show the first four squares in a sequence of squares which are subdivided in half. The area of the shaded square A is $\frac{1}{4}$.

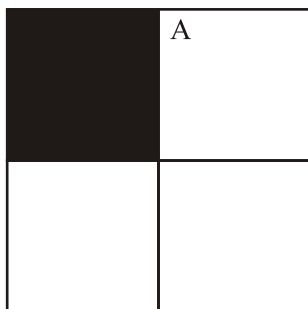


Diagram 1

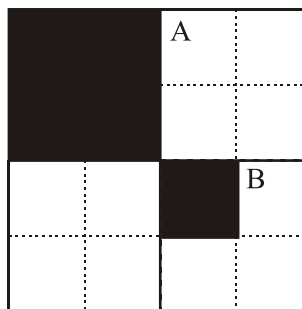


Diagram 2

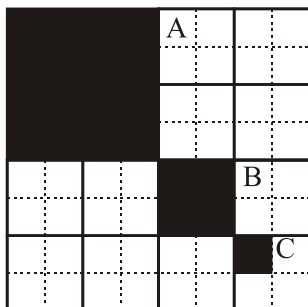


Diagram 3

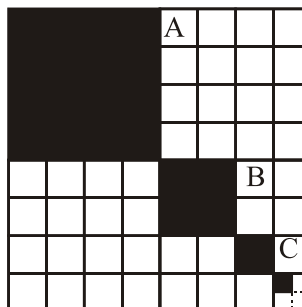


Diagram 4

- (a) (i) Find the area of square B and of square C.
 (ii) Show that the areas of squares A, B and C are in geometric progression.
 (iii) Write down the common ratio of the progression.

(5)

- (b) (i) Find the **total** area shaded in diagram 2.
 (ii) Find the **total** area shaded in the 8th diagram of this sequence.
 Give your answer correct to six significant figures.

(4)

- (c) The dividing and shading process illustrated is continued indefinitely.
 Find the total area shaded.

(2)

(Total 11 marks)

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20. A company offers its employees a choice of two salary schemes A and B over a period of 10 years.

Scheme A offers a starting salary of \$11000 in the first year and then an annual increase of \$400 per year.

- (a) (i) Write down the salary paid in the second year and in the third year.
(ii) Calculate the **total** (amount of) salary paid over ten years.

(3)

Scheme B offers a starting salary of \$10000 dollars in the first year and then an annual increase of 7% of the previous year's salary.

- (b) (i) Write down the salary paid in the second year and in the third year.
(ii) Calculate the salary paid in the tenth year.

(4)

- (c) Arturo works for n complete years under scheme A. Bill works for n complete years under scheme B. Find the minimum number of years so that the total earned by Bill exceeds the total earned by Arturo.

(4)

(Total 11 marks)