

Vector Applications

We can use vectors when we have vector quantities such as displacements and velocities.

If $\begin{pmatrix} a \\ b \end{pmatrix}$ is the **velocity vector** of a moving object, then it is travelling at a speed of $\left| \begin{pmatrix} a \\ b \end{pmatrix} \right| = \sqrt{a^2 + b^2}$ in the direction $\begin{pmatrix} a \\ b \end{pmatrix}$.

Magnitude of a velocity vector tells you how fast the object moves!

Find the speed of the object with velocity vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$$5 \text{ km/h}$$

The object speeds up to 20 km/h, what is its velocity vector?

$$\begin{pmatrix} 16 \\ -12 \end{pmatrix}$$

$$\frac{20}{5} \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

- 1 Each of the following vector equations represents the path of a moving object. t is measured in seconds and $t \geq 0$. Distances are measured in metres. In each case, find:
- i the initial position ii the velocity vector iii the speed of the object.

a $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + t \begin{pmatrix} 12 \\ 5 \end{pmatrix}$

i) $(-4, 3)$

ii) $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$

iii) 13 m/s

b $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

i) $(0, -6)$

ii) $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

iii) 5 m/s

2 Find the velocity vector of a speed boat moving parallel to:

a $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ with a speed of 150 km h^{-1}

b $\begin{pmatrix} 24 \\ 7 \end{pmatrix}$ with a speed of 12.5 km h^{-1}

c $2\mathbf{i} + \mathbf{j}$ with a speed of 50 km h^{-1}

a) $\begin{pmatrix} 120 \\ -90 \end{pmatrix}$

b) $\begin{pmatrix} 12 \\ 3.5 \end{pmatrix}$

c) $\begin{pmatrix} \frac{100}{\sqrt{5}} \\ \frac{50}{\sqrt{5}} \end{pmatrix}$

- 3 Find the velocity vector of a swooping eagle moving in the direction $-2\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}$ with a speed of 90 km h^{-1} .

$$\begin{pmatrix} -12 \\ 30 \\ -84 \end{pmatrix}$$

$$\frac{90}{15} \begin{pmatrix} -2 \\ 5 \\ -14 \end{pmatrix}$$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ is the vector equation of the path of an object.

t is the time in seconds, $t \geq 0$. The distance units are metres.

- a** Find the object's initial position
- b** Find the velocity vector of the object
- c** Find the object's speed.
- d** If the object continues in the same direction but increases its speed to 30 m s^{-1} , state its new velocity vector.

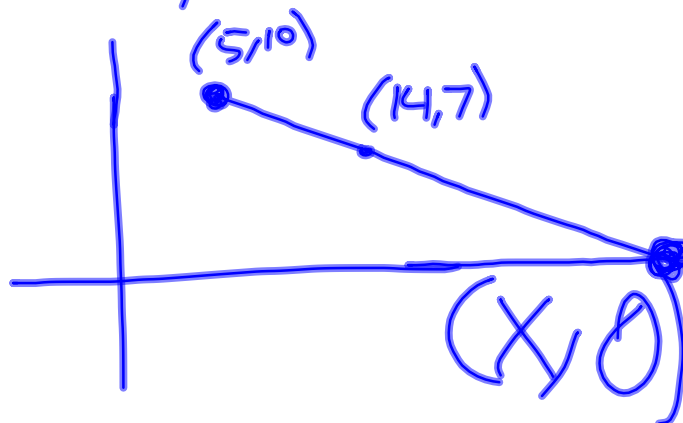
An object is initially at $(5, 10)$ and moves with velocity vector $3\mathbf{i} - \mathbf{j}$. Find:

- a the position of the object at any time t where t is in minutes
- b the position at $t = 3$
- c the time when the object is due east of $(0, 0)$.

$$a) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$b) (14, 7)$$

$$c) t = 10$$



$$\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

4 A helicopter at A(6, 9, 3) moves with constant velocity in a straight line. 10 minutes later it is at B(3, 10, 2.5). If distances are in kilometres, find:

- a \vec{AB}
- b the helicopter's speed in km/h
- c the equation of the straight line
- d the time taken until the helicopter lands on the helipad where $z = 0$.

a) $\begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}$

b) $\approx 3.2 \text{ km}/10 \text{ min}$
 19.2 km/h

c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}$

$\vec{v} = \begin{pmatrix} -18 \\ 3 \\ 3 \end{pmatrix}$

d) 1 hr

$t = 6$

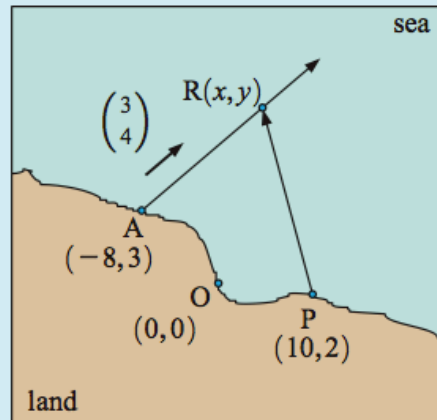
The position vector of a boat, A, t hours after it leaves a harbour is given by $\mathbf{r}_1 = t \begin{pmatrix} 30 \\ 15 \end{pmatrix}$. A second boat, B, is passing near the harbour. Its position vector at time t is given by $\mathbf{r}_2 = \begin{pmatrix} 50 \\ 5 \end{pmatrix} + t \begin{pmatrix} 10 \\ 10 \end{pmatrix}$.

- a** How far apart are the two boats at the time the first boat leaves the harbour?
- b** How fast is each boat traveling?
- c** Are the boats in danger of colliding if one of the boats does not change course?



On the map shown, distances are measured in kilometres. Ship R moves in the direction $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ at a speed of 10 km h^{-1} .

- Find an expression for the position of the ship in terms of t , the number of hours after leaving port A.
- Find the time when the ship is closest to port P(10, 2).



1 An ocean liner is at $(6, -6)$, cruising at 10 km h^{-1} in the direction $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

A fishing boat is anchored at $(0, 0)$. Distances are in kilometres.

- a** Find, in terms of \mathbf{i} and \mathbf{j} , the original position vector of the liner from the fishing boat.
- b** Write an expression for the position vector of the liner at any time t hours after it has sailed from $(6, -6)$.
- c** Find when the liner is due east of the fishing boat.
- d** Find the time and position of the liner when it is nearest to the fishing boat.

Homework

Vector Applications Worksheet

Finish examples from Notes