Vector Applications

We can use vectors when we have vector quantities such as displacements and velocities.

If $\begin{pmatrix} a \\ b \end{pmatrix}$ is the **velocity vector** of a moving object, then it is travelling at a speed of $\left|\begin{pmatrix} a \\ b \end{pmatrix}\right| = \sqrt{a^2+b^2}$ in the direction $\begin{pmatrix} a \\ b \end{pmatrix}$.

Magnitude of a velocity vector tells you how fast the object moves!

Find the speed of the object with velocity vector $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

5 km/h

The object speeds up to 20 km/h, what is its velocity vector?

$\begin{pmatrix} 16 \\ -12 \end{pmatrix}$ $\frac{20}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

1 Each of the following vector equations represents the path of a moving object. t is measured in seconds and $t \geqslant 0$. Distances are measured in metres. In each case, find:

i the initial position **ii** the velocity vector **iii** the speed of the object.

a $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 12 \\ 5 \end{pmatrix}$

i) $(-4, 3)$

ii) $\begin{pmatrix} 12 \\ 5 \end{pmatrix}$

iii) 13 m/s

ms^{-1}

b $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \end{pmatrix} + t\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

i) $(0, -6)$

ii) $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

iii) 5 m/s

2 Find the velocity vector of a speed boat moving parallel to:

a $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ with a speed of 150 km h^{-1}

b $\begin{pmatrix} 24 \\ 7 \end{pmatrix}$ with a speed of 12.5 km h^{-1}

c $2\mathbf{i} + \mathbf{j}$ with a speed of 50 km h^{-1}

a) $\begin{pmatrix} 120 \\ -90 \end{pmatrix}$ b) $\begin{pmatrix} 12 \\ 3.5 \end{pmatrix}$ c) $\begin{pmatrix} \frac{100}{\sqrt{5}} \\ \frac{50}{\sqrt{5}} \end{pmatrix}$

3 Find the velocity vector of a swooping eagle moving in the direction $-2\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}$ with a speed of 90 km h^{-1}.

$$\begin{pmatrix} -12 \\ 30 \\ -84 \end{pmatrix}$$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 5 \end{pmatrix} + t \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ is the vector equation of the path of an object.

t is the time in seconds, $t \geqslant 0$. The distance units are metres.

a Find the object's initial position

b Find the velocity vector of the object

c Find the object's speed.

d If the object continues in the same direction but increases its speed to 30 m s^{-1}, state its new velocity vector.

a) $(7, 5)$

b) $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$

c) 10m/s

d) $\begin{pmatrix} 18 \\ -24 \end{pmatrix}$

An object is initially at (5, 10) and moves with velocity vector $3\mathbf{i} - \mathbf{j}$. Find:

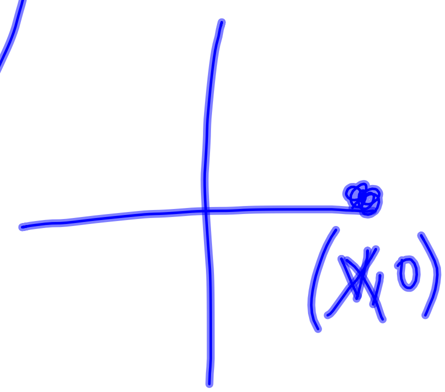
a the position of the object at any time t where t is in minutes

b the position at $t = 3$

c the time when the object is due east of (0, 0).

a) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

b) (14, 7)



c) $\begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$x = 5 + 3t \quad 0 = 10 - t$

$\boxed{t = 10 \text{ min}}$

4 A helicopter at A(6, 9, 3) moves with constant velocity in a straight line. 10 minutes later it is at B(3, 10, 2.5). If distances are in kilometres, find:

a $\overrightarrow{AB}$

b the helicopter's speed

c the equation of the straight line

d the time taken until the helicopter lands on the helipad where $z = 0$.

a) $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}$

velocity vector per hr.

$\begin{pmatrix} -18 \\ 6 \\ -3 \end{pmatrix}$

b) 3.2 km/10min

≈ 19.2 km/h

c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ 3 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \\ -0.5 \end{pmatrix}$

d) $0 = 3 - 0.5t$

$6 = t$

$\boxed{1\text{ hr}}$

The position vector of a boat, A, t hours after it leaves a harbour is given by $\mathbf{r}_1 = t\begin{pmatrix}30\\15\end{pmatrix}$. A second boat, B, is passing near the harbour. Its position vector at time t is given by $\mathbf{r}_2 = \begin{pmatrix}50\\5\end{pmatrix} + t\begin{pmatrix}10\\10\end{pmatrix}$.

a How far apart are the two boats at the time the first boat leaves the harbour?

b How fast is each boat traveling?

c Are the boats in danger of colliding if one of the boats does not change course?

a) ≈ 50.2

b) A: ≈ 33.5 units/hr

B: ≈ 14.1 units/hr

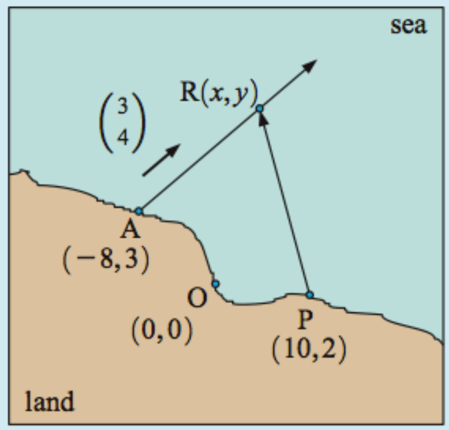
c) $30t = 50 + 10t$ $\quad$ $15t = 5 + 10t$

$t = \frac{5}{2}$ $\quad$ $t = 1$

On the map shown, distances are measured in kilometres. Ship R moves in the direction $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ at a speed of $10\ \text{km h}^{-1}$.

a Find an expression for the position of the ship in terms of t, the number of hours after leaving port A.

b Find the time when the ship is closest to port P(10, 2).



1 An ocean liner is at (6, −6), cruising at 10 $\text{km}\,\text{h}^{-1}$ in the direction $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

A fishing boat is anchored at (0, 0). Distances are in kilometres.

a Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the original position vector of the liner from the fishing boat.

b Write an expression for the position vector of the liner at any time t hours after it has sailed from (6, −6).

c Find when the liner is due east of the fishing boat.

d Find the time and position of the liner when it is nearest to the fishing boat.

Homework

Vector Applications Worksheet

Finish examples from Notes