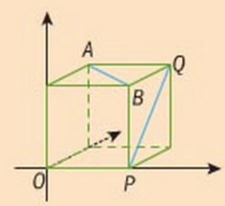
We can also figure out where two vector lines intersect.

In three dimensions, two lines will either

1. **intersect** – if the value of the variables is consistent in all three equations
2. **be parallel** – they will have direction vectors that are scalar multiples of each other
3. **be skew** – if the lines are not parallel and the values are not consistent so the lines do not intersect.



AB and *PQ* are skew – they never meet.

Find the point of intersection of the line L_1 and L_2 whose vector equations are $r_1 = \begin{pmatrix}2\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\1\end{pmatrix}$ and $r_2 = \begin{pmatrix}5\\-2\end{pmatrix} + \mu\begin{pmatrix}1\\-2\end{pmatrix}$

Write out the components, equate and solve

$2 + 2\lambda = 5 + \mu$

$2\lambda - \mu = 3$

$3 + \lambda = -2 - 2\mu$

$\lambda + 2\mu = -5$

$4\lambda - 2\mu = 6$

$+ \quad \lambda + 2\mu = -5$

$5\lambda = 1$

$\lambda = \frac{1}{5}$

$\frac{1}{5} + 2\mu = -5$

$2\mu = -\frac{26}{5}$

$\mu = -\frac{13}{5}$

$\begin{pmatrix}2\\3\end{pmatrix} + \frac{1}{5}\begin{pmatrix}2\\1\end{pmatrix}$

$\begin{pmatrix}\frac{12}{5}\\\frac{16}{5}\end{pmatrix}$

$\begin{pmatrix}5\\-2\end{pmatrix} - \frac{13}{5}\begin{pmatrix}1\\-2\end{pmatrix}$

$\begin{pmatrix}\frac{12}{5}\\\frac{16}{5}\end{pmatrix}$

$\boxed{\left(\frac{12}{5}, \frac{16}{5}\right)}$

Two lines have equations $\mathbf{r}_1 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + t\begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}$.

Show that the lines intersect and find the coordinates of the point of intersection.

Find a vector equation of the line passing through the origin that also passes through the point of intersection of the lines

$v = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $u = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \end{pmatrix}$

Line L passes through points A(1,2,-1) and B(11,-2,-7) while Line M passes through points C(2,-1,-3) and D(9,-10,3). Show that L and M are skew lines.

$\vec{L} = \begin{pmatrix}1\\2\\-1\end{pmatrix} + t\begin{pmatrix}10\\-4\\-6\end{pmatrix}$ $\vec{M} = \begin{pmatrix}2\\-1\\-3\end{pmatrix} + s\begin{pmatrix}7\\-9\\6\end{pmatrix}$

$\begin{pmatrix}10\\-4\\-6\end{pmatrix} = k\begin{pmatrix}7\\-9\\6\end{pmatrix}$

$10 = 7k$ $k = \frac{10}{7}$

$-4 = -9k$ $k = \frac{4}{9}$

$\frac{10}{7} \neq \frac{4}{9}$ $\therefore \vec{L} + \vec{M}$ are not parallel

$1 + 10t = 2 + 7s$ $2 - 4t = -1 - 9s$ $-1 - 6t = -3 + 6s$

$1 + 10\left(\frac{6}{13}\right) = 2 + 7\left(-\frac{5}{39}\right)$ $t = \frac{6}{13}$ $s = -\frac{5}{39}$

(78, −35)

$\frac{73}{13} \neq \frac{43}{39}$ $\therefore \vec{L} + \vec{M}$ don't intersect

Therefore $\vec{L} + \vec{M}$ are skew.

Are the lines $m = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$ and $n = \begin{pmatrix} -2 \\ -7 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$ parallel?

Find the point of intersection of these lines.

What do you conclude?

IB Problems - Vectors

1. Consider the vectors $\boldsymbol{u} = 2\boldsymbol{i} + 3\boldsymbol{j} - \boldsymbol{k}$ and $\boldsymbol{v} = 4\boldsymbol{i} + \boldsymbol{j} - p\boldsymbol{k}$.

(a) Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{v}$ find the value of p.

(b) Given that $q|\mathbf{u}| = 14$, find the value of q.

(Total 6 marks)

IB Problems - Vectors

2. Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AC}$.

(3)

(b) (i) Find $\overrightarrow{AB} \bullet \overrightarrow{AC}$.

(ii) Find the sine of the angle between $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(3)

(Total 6 marks)

IB Problems - Vectors

3. Two lines L_1 and L_2 are given by $\boldsymbol{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s\begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a) Let θ be the acute angle between L_1 and L_2. Show that $\cos\theta = \frac{52}{140}$.

(5)

(b) (i) P is the point on L_1 when $s = 1$. Find the position vector of P.

(ii) Show that P is also on L_2.

(8)

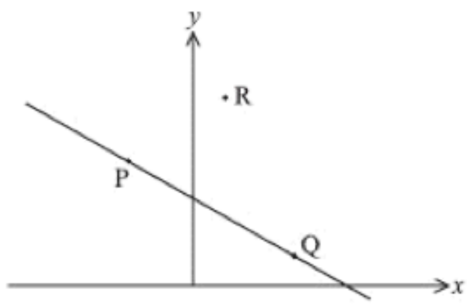
(c) A third line L_3 has direction vector $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$. If L_1 and L_3 are parallel, find the value of x.

(3)

(Total 16 marks)

IB Problems - Vectors

4. The points P(−2, 4), Q (3, 1) and R (1, 6) are shown in the diagram below.



(a) Find the vector $\overrightarrow{PQ}$.

(b) Find a vector equation for the line through R parallel to the line (PQ).

(Total 6 marks)

IB Problems - Vectors KEY

1. (a) $\boldsymbol{u} \bullet \boldsymbol{v} = 8 + 3 + p$ (A1)

For equating scalar product equal to zero (M1)

$8 + 3 + p = 0$

$p = -11$ A1 N3

(b) $|\boldsymbol{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}$, 3.74 (M1)

$q\sqrt{14} = 14$ A1

$q = \sqrt{14}$ $(= 3.74)$ A1 N2

[6]

2.	(a)	(i)	evidence of combining vectors	(M1)	
			e.g. $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$		
			$\overrightarrow{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$	A1	N2
		(ii)	$\overrightarrow{AC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$	A1	N1
	(b)	(i)	$\overrightarrow{AB} \bullet \overrightarrow{AC} = (-2)(3) + (-3)(-2) = 0$	A1	N1
		(ii)	scalar product $0 = \Rightarrow$ perpendicular, $\theta = 90°$	(R1)	
			$\sin \theta = 1$	A1	N2
					[6]

3. (a) Using direction vectors $\boldsymbol{u} = \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

$|\boldsymbol{u}| = \sqrt{4+36+100} = \sqrt{140}, |\boldsymbol{v}| = \sqrt{36+100+4} = \sqrt{140}$ A1A1

$\boldsymbol{u} \bullet \boldsymbol{v} = 12 + 60 - 20 = 52$ A1

$\cos\theta = \dfrac{52}{\sqrt{140}\sqrt{140}}$ A1

$= \dfrac{52}{140}$ AG N0

(b) (i) For substituting $s = 1$ (M1)

Correct calculations (A1)

$9 + 1(-2) = 7,\ 4 + 1(6) = 10,\ -6 + 1(10) = 4$

position vector of P is $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix}$ A1 N3

(ii) For substituting into the equation $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

For one correct equation A1

e.g. $7 = 1 - 6t$

Solving gives $t = -1$ A1

verify for second coordinate, $10 = 20 + (-1)(10)$ A1

verify for third coordinate, $4 = 2 + (-1)(-2)$ A1

Thus, P is also on L_2. AG N0

(c) $k\begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$ (M1)

$-2k = 6$

$k = -3$ A1

$x = -3 \times 6 = -18$ A1 N2

[16]

4. (a) $\vec{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ A1A1 N2

(b) Using $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t\begin{pmatrix} 5 \\ -3 \end{pmatrix}$$ A2A1A1 N4

[6]

Homework

Chapter 12.4

12K: 1 - 8