

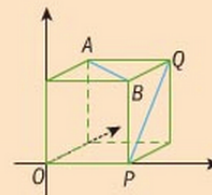
We can also figure out where two vector lines intersect.

In three dimensions,
two lines will either

1 intersect – if the value of the variables is consistent in all three equations

2 be parallel – they will have direction vectors that are scalar multiples of each other

3 be skew – if the lines are not parallel and the values are not consistent so the lines do not intersect.



AB and PQ are skew
– they never meet.

Find the point of intersection of the line L_1 and L_2 whose vector equations are $r_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $r_2 = \begin{pmatrix} 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Write out the components,
equate and solve

$$2 + 2\lambda = 5 + \mu$$

$$2\lambda - \mu = 3$$

$$4\lambda - 2\mu = 6$$

$$+ \quad \lambda + 2\mu = -5$$

$$\hline 5\lambda = 1$$

$$\lambda = \frac{1}{5}$$

$$3 + \lambda = -2 - 2\mu$$

$$\lambda + 2\mu = -5$$

$$\frac{1}{5} + 2\mu = -5$$

$$2\mu = -\frac{26}{5}$$

$$\mu = -\frac{13}{5}$$

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \frac{1}{5} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{12}{5} \\ \frac{16}{5} \end{pmatrix}$$

$$\boxed{\left(\frac{12}{5}, \frac{16}{5} \right)}$$

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \frac{13}{5} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

$$\begin{pmatrix} \frac{12}{5} \\ \frac{16}{5} \end{pmatrix}$$

Two lines have equations $\mathbf{r}_1 = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 6 \\ 2 \\ 0 \end{pmatrix} + t \begin{pmatrix} 0 \\ 4 \\ 8 \end{pmatrix}$.

Show that the lines intersect and find the coordinates of the point of intersection.

Find a vector equation of the line passing through the origin that also passes through the point of intersection of the lines

$$\underline{v} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \text{and} \quad \underline{u} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \end{pmatrix}$$

$$\lambda = \frac{2}{7}$$
$$\mu = \frac{13}{7}$$

$$\underline{r} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} \frac{19}{7} \\ \frac{20}{7} \end{pmatrix}$$

Line L passes through points A(1,2,-1) and B(11,-2,-7) while Line M passes through points C(2,-1,-3) and D(9,-10,3). Show that L and M are skew lines.

$$L = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + s \begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix} \quad M = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} + t \begin{pmatrix} 7 \\ -9 \\ 6 \end{pmatrix}$$

$$1 + 10s = 2 + 7t \quad \left\{ \begin{array}{l} 2 - 4s = -1 - 9t \\ -1 - 6s = -3 + 6t \end{array} \right.$$

$$1 + 10\left(\frac{6}{13}\right) = 2 + 7\left(\frac{-5}{39}\right) \quad s = \frac{6}{13} \quad t = \frac{-5}{39}$$

$$\frac{73}{13} \neq \frac{43}{39}$$

$\therefore L + M$ don't intersect

$$\begin{pmatrix} 10 \\ -4 \\ -6 \end{pmatrix} = k \begin{pmatrix} 7 \\ -9 \\ 6 \end{pmatrix} \quad \begin{array}{l} 10 = 7k \\ \frac{10}{7} = k \end{array} \quad \begin{array}{l} -4 = -9k \\ \frac{4}{9} = k \end{array}$$

$$\frac{10}{7} \neq \frac{4}{9}$$

$\therefore L + M$ are not parallel

Therefore $L + M$ are skew!

Are the lines $m = \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}$ and $n = \begin{pmatrix} -2 \\ -7 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$ parallel?

Find the point of intersection of these lines.

What do you conclude?

IB Problems - Vectors

1. Consider the vectors $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 4\mathbf{i} + \mathbf{j} - p\mathbf{k}$.
- (a) Given that \mathbf{u} is perpendicular to \mathbf{v} find the value of p .
- (b) Given that $q|\mathbf{u}|=14$, find the value of q .

(Total 6 marks)

IB Problems - Vectors

2. Consider the points A(5, 8), B(3, 5) and C(8, 6).

(a) Find

(i) \overrightarrow{AB} ;

(ii) \overrightarrow{AC} .

(3)

(b) (i) Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

(ii) Find the sine of the angle between \overrightarrow{AB} and \overrightarrow{AC} .

(3)

(Total 6 marks)

IB Problems - Vectors

3. Two lines L_1 and L_2 are given by $\mathbf{r}_1 = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} + s \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\mathbf{r}_2 = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$.

(a) Let θ be the acute angle between L_1 and L_2 . Show that $\cos\theta = \frac{52}{140}$.

(5)

(b) (i) P is the point on L_1 when $s = 1$. Find the position vector of P.

(ii) Show that P is also on L_2 .

(8)

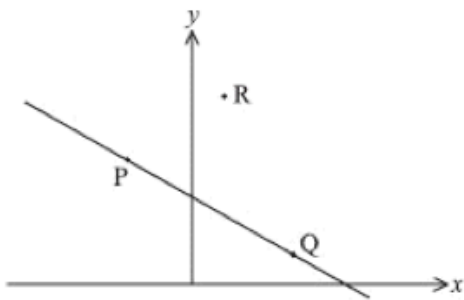
(c) A third line L_3 has direction vector $\begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$. If L_1 and L_3 are parallel, find the value of x .

(3)

(Total 16 marks)

IB Problems - Vectors

4. The points $P(-2, 4)$, $Q(3, 1)$ and $R(1, 6)$ are shown in the diagram below.



- (a) Find the vector \overrightarrow{PQ} .
- (b) Find a vector equation for the line through R parallel to the line (PQ).

(Total 6 marks)

IB Problems - Vectors KEY

1. (a) $\mathbf{u} \cdot \mathbf{v} = 8 + 3 + p$ (A1)
For equating scalar product equal to zero (M1)
 $8 + 3 + p = 0$
 $p = -11$ A1 N3
- (b) $|\mathbf{u}| = \sqrt{2^2 + 3^2 + (-1)^2} = \sqrt{14}, 3.74$ (M1)
 $q\sqrt{14} = 14$ A1
 $q = \sqrt{14} (=3.74)$ A1 N2

[6]

2. (a) (i) evidence of combining vectors (M1)
e.g. $\vec{AB} = \vec{OB} - \vec{OA}$
 $\vec{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ A1 N2
- (ii) $\vec{AC} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ A1 N1
- (b) (i) $\vec{AB} \cdot \vec{AC} = (-2)(3) + (-3)(-2) = 0$ A1 N1
- (ii) scalar product $0 = \Rightarrow$ perpendicular, $\theta = 90^\circ$ (R1)
 $\sin \theta = 1$ A1 N2
- [6]**

3. (a) Using direction vectors $\mathbf{u} = \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

$|\mathbf{u}| = \sqrt{4 + 36 + 100} = \sqrt{140}$, $|\mathbf{v}| = \sqrt{36 + 100 + 4} = \sqrt{140}$ A1A1

$\mathbf{u} \cdot \mathbf{v} = 12 + 60 - 20 = 52$ A1

$\cos \theta = \frac{52}{\sqrt{140}\sqrt{140}}$ A1

$= \frac{52}{140}$ AG N0

(b) (i) For substituting $s = 1$ (M1)
 Correct calculations (A1)
 $9 + 1(-2) = 7$, $4 + 1(6) = 10$, $-6 + 1(10) = 4$

position vector of P is $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix}$ A1 N3

(ii) For substituting into the equation $\begin{pmatrix} 7 \\ 10 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 20 \\ 2 \end{pmatrix} + t \begin{pmatrix} -6 \\ 10 \\ -2 \end{pmatrix}$ (M1)

For one correct equation A1
 e.g. $7 = 1 - 6t$
 Solving gives $t = -1$ A1

verify for second coordinate, $10 = 20 + (-1)(10)$ A1
 verify for third coordinate, $4 = 2 + (-1)(-2)$ A1
 Thus, P is also on L_2 . AG N0

(c) $k \begin{pmatrix} -2 \\ 6 \\ 10 \end{pmatrix} = \begin{pmatrix} 6 \\ x \\ -30 \end{pmatrix}$ (M1)

$-2k = 6$
 $k = -3$ A1
 $x = -3 \times 6 = -18$ A1 N2

[16]

4. (a) $\vec{PQ} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

A1A1 N2

(b) Using $r = a + tb$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \end{pmatrix} + t \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

A2A1A1 N4

[6]

Homework
Chapter 12.4
12K: 1 - 8