

Vector Equation of a Line

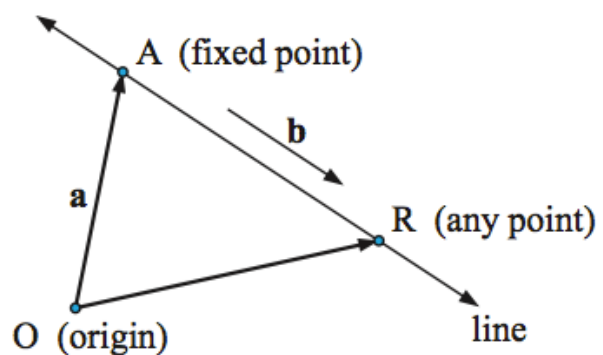
A line passes through point A (where A had position vector \mathbf{a}) and the line is parallel to vector \mathbf{b} .

R is any point on the line, then \overrightarrow{AR} is parallel to \mathbf{b} .

$$\overrightarrow{AR} \parallel \mathbf{b}, \quad \overrightarrow{AR} = t\mathbf{b} \text{ for some } t \in \mathbb{R}$$

Using vector addition we can see $\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR}$.

$$\therefore \mathbf{r} = \mathbf{a} + t\mathbf{b}$$



→ The **vector equation** of a line is given by $\mathbf{r} = \mathbf{a} + t\mathbf{b}$ where \mathbf{r} is the general position vector of a point on the line, \mathbf{a} is a given position vector of a point on the line and \mathbf{b} is a **direction vector** parallel to the line. t is called the parameter.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Find the vector equation through $A(1, 5)$ with direction $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$\vec{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Find the vector equation through $(1, -2, 3)$ in the direction $4\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$.

$$\vec{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 4 \\ 5 \\ -6 \end{pmatrix}$$

Find the vector equation of the line through $A(2, -1, 4)$ and $B(-1, 0, 2)$.

$$\vec{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix}$$

Find the angle between the lines:

$$L_1: \mathbf{r}_1 = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + s \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \quad \text{and} \quad L_2: \mathbf{r}_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix}.$$

$$\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = k \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \therefore \text{parallel} \therefore \theta = 0^\circ$$

Find the angle between the lines:

$$L_1: \mathbf{r}_1 = \begin{pmatrix} 8 \\ 9 \\ 10 \end{pmatrix} + s \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \quad \text{and} \quad L_2: \mathbf{r}_2 = \begin{pmatrix} 15 \\ 29 \\ 5 \end{pmatrix} + t \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix}$$

$$\theta = 151^\circ$$

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$$\cos^{-1} \left(\frac{-154}{\sqrt{314}\sqrt{98}} \right)$$

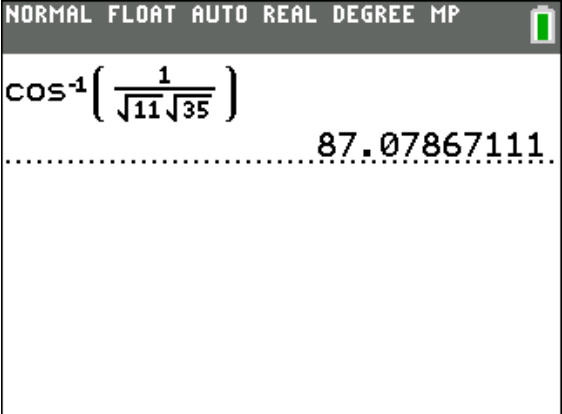
..... 151.3895403

- a** Find the vector equation of the line through $(1, -1, 3)$ and parallel to the vector $-\mathbf{i} + 3\mathbf{j} - \mathbf{k}$
- b** Find the vector equation of the line through the points $A(1, 0, -4)$ and $B(-2, 1, 1)$.
- c** Find the acute angle between these two lines.

$$a) \vec{r} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

$$b) \vec{r} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$$

$$c) \theta = 87.1^\circ$$



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$$\cos^{-1}\left(\frac{1}{\sqrt{11}\sqrt{35}}\right)$$

.....87.07867111

You can think of each part of the vector equation as separate components.

For example consider the following vector equation:

$$\mathbf{r} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} -1 \\ 3 \end{pmatrix} \text{ where } \mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Therefore:

$$x = 1 - t$$

$$y = 5 + 3t$$

This is called parametric form and can be useful when solving!

This extends to 3D too!

Another example:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 20 \end{pmatrix} + t \begin{pmatrix} 4 \\ -3 \end{pmatrix} \text{ then } x = 2 + 4t \text{ and } y = 20 - 3t.$$

Does $(3, -2)$ lie on the line with vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \end{pmatrix}$?

Yes, when $t=1$

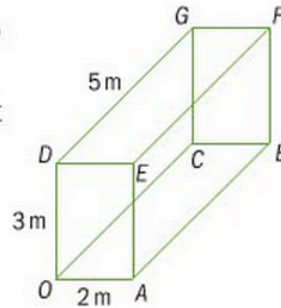
$$3 = 2 + t \quad -2 = 1 - 3t$$

$(k, 4)$ lies on the line $r = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ Find k .

$$k = \frac{1}{5}$$

10 The figure shows a cuboid in which $OA = 2$ m, $OC = 5$ m and $OD = 3$ m.

Take O as the origin and unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} in the direction OA , OC and OD respectively.



a Express these vectors in terms of the unit vectors.

i \vec{OF} **ii** \vec{AG}

b Calculate the value of

i $|\vec{OF}|$ **ii** $|\vec{AG}|$

iii Find the scalar product of \vec{OF} and \vec{AG} .

c Hence find the angle between the diagonals OF and AG .

a) i) $\vec{OF} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$

ii) $\vec{AG} = -2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$

b) i) $|\vec{OF}| = \sqrt{38}$ ii) $|\vec{AG}| = \sqrt{38}$

iii) $\vec{OF} \cdot \vec{AG} = 30$

c) $\theta = 37.9^\circ$

Homework

Chapter 12.4

12J: 1 - 9, 11