Warm-up

If $\mathbf{a} = \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ find:

a $|\mathbf{a}|$ **b** $|\mathbf{b}|$ **c** $|\mathbf{b}+\mathbf{c}|$ **d** $|\mathbf{a}-\mathbf{c}|$ **e** $|\mathbf{a}|\mathbf{b}$ **f** $\frac{1}{|\mathbf{a}|}\mathbf{a}$

$\sqrt{11}$ $\sqrt{14}$ $\sqrt{38}$ $\sqrt{3}$ $\begin{pmatrix} \sqrt{11} \\ -3\sqrt{11} \\ 2\sqrt{11} \end{pmatrix}$ $\begin{pmatrix} -\frac{1}{\sqrt{11}} \\ \frac{1}{\sqrt{11}} \\ \frac{3}{\sqrt{11}} \end{pmatrix}$

$\vec{b}+\vec{c} = \left|\begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}\right|$ $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

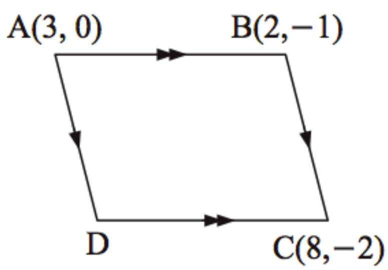
Geometrical Proofs with Vectors

Geometrical proofs

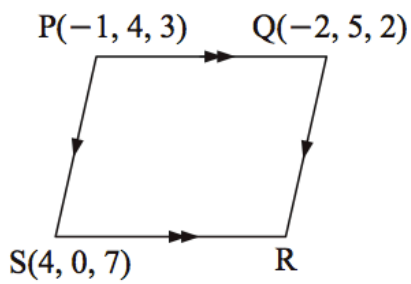
When you are not given specific vectors you can still use vector addition, subtraction and scalar multiples to deduce some geometrical results.

Use vector methods to find the remaining vertex of:

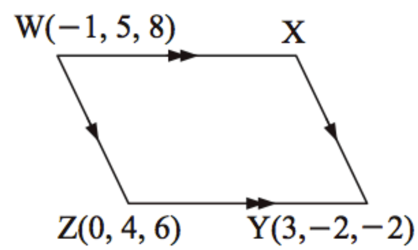
a



b



c

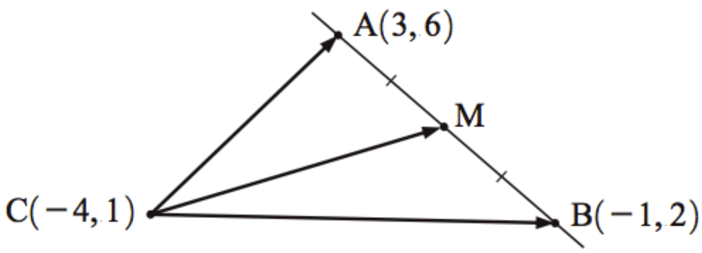


$\overrightarrow{AD} = \overrightarrow{BC}$

D(9, −1)

R(3, 1, 6)

X(2, −1, 0)



a Find the coordinates of M.

b Find vectors $\overrightarrow{CA}$, $\overrightarrow{CM}$ and $\overrightarrow{CB}$.

c Verify that $\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CA} + \frac{1}{2}\overrightarrow{CB}$.

a) $(1, 4)$

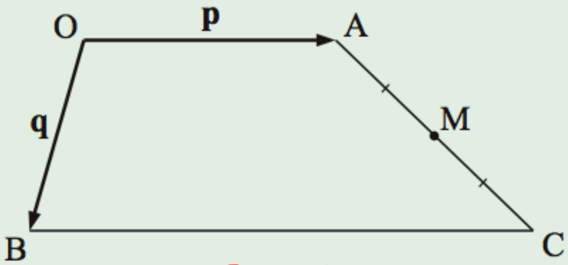
b) $\overrightarrow{CA} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}$

$\overrightarrow{CM} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$

$\overrightarrow{CB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

c. $\begin{pmatrix} 5 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{7}{2} \\ \frac{5}{2} \end{pmatrix} + \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$

$\begin{pmatrix} \frac{10}{2} \\ \frac{6}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$



[BC] is parallel to [OA] and is twice its length. Find, in terms of **p** and **q**, vector expressions for: **a** $\overrightarrow{AC}$ **b** $\overrightarrow{OM}$.

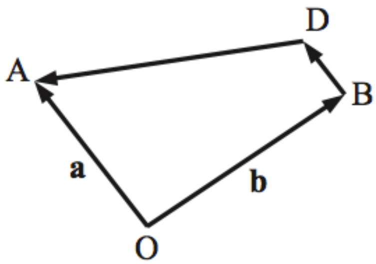
~~a = p + q~~

$p + q$

b) $\vec{p} + \frac{1}{2}(\vec{p} + \vec{q})$

$p + \frac{1}{2}p + \frac{1}{2}q$

$1.5p + \frac{1}{2}q$



In the given figure [BD] is parallel to [OA] and half its length. Find, in terms of **a** and **b**, vector expressions for:

- **a** $\overrightarrow{BD}$
- **b** $\overrightarrow{AB}$
- **c** $\overrightarrow{BA}$
- **d** $\overrightarrow{OD}$
- **e** $\overrightarrow{AD}$
- **f** $\overrightarrow{DA}$

a) $\frac{1}{2}\underline{a}$

b) $-\underline{a}+\underline{b}$

c) $\underline{a}-\underline{b}$

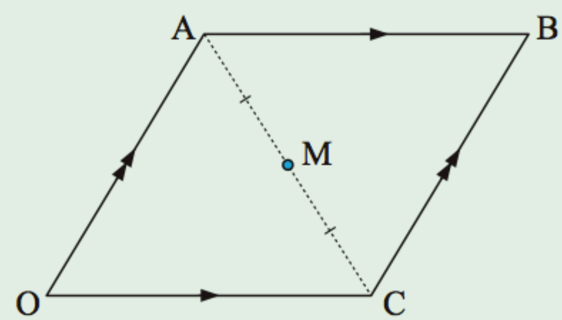
d) $\underline{b}+\frac{1}{2}\underline{a}$

e) $-\frac{a}{2}+b$

f) $\frac{1}{2}a - b$

In this question you may **not** assume any diagonal properties of parallelograms.

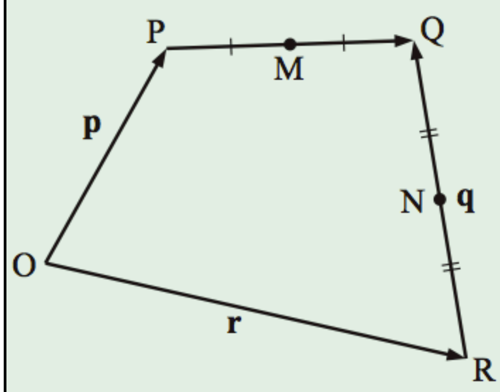
OABC is a parallelogram with $\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$. M is the midpoint of [AC].



a Find in terms of $\mathbf{p}$ and $\mathbf{q}$:

i $\overrightarrow{OB}$ **ii** $\overrightarrow{OM}$

b Using **a** only, show that O, M and B are collinear, and that M is the midpoint of [OB].

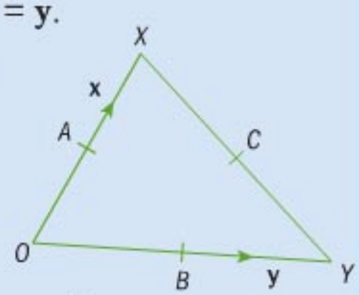


In the figure alongside, $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{RQ} = \mathbf{q}$.

If M and N are midpoints of the sides as shown, find in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$:

a $\overrightarrow{OQ}$ **b** $\overrightarrow{PQ}$ **c** $\overrightarrow{ON}$ **d** $\overrightarrow{MN}$

In triangle OXY, A, B and C are the midpoints of OX, OY and XY respectively, $\overrightarrow{OX} = \mathbf{x}$ and $\overrightarrow{OY} = \mathbf{y}$.



a Find expressions for $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{XY}$, $\overrightarrow{OC}$ and $\overrightarrow{CO}$ in terms of $\mathbf{x}$ and $\mathbf{y}$.

b Find an expression for $\overrightarrow{AB}$ in terms of $\mathbf{x}$ and $\mathbf{y}$. What is the relationship between the line XY and the line AB?

c P is the point such that $\overrightarrow{OP} = \overrightarrow{OX} + \frac{2}{3}\overrightarrow{XP}$. Find $\overrightarrow{OP}$.

d What can you conclude about the position of P?

Homework

Chapter 12.2

12H: 1-5

Review for Quiz

- Resultant Vectors
- Using vectors to show/use Geometric properties
- Vector addition/subtraction
- Magnitude of a vector
- Unit vectors
- Parallel vectors