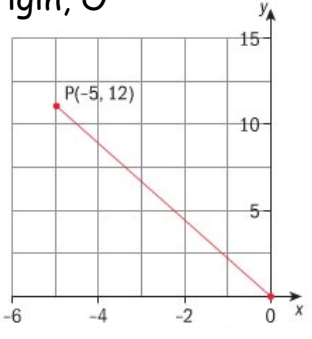
Review of equal vectors

Find a, b, and c if $\begin{pmatrix} a-3 \\ b-2 \\ c-1 \end{pmatrix} = \begin{pmatrix} 1-a \\ -b \\ -3-c \end{pmatrix}$.

Position vectors

Position of a point relative to the origin, O

The point P with coordinates $(-5, 12)$ has position vector $\overrightarrow{OP} = \begin{pmatrix} -5 \\ 12 \end{pmatrix} = -5\mathbf{i} + 12\mathbf{j}$.



→ The point P with coordinates (x, y) has position vector

$\overrightarrow{OP} = \begin{pmatrix} x \\ y \end{pmatrix} = x\mathbf{i} + y\mathbf{j}$.

Resultant vectors

The final vector when more than one vector is combined

For example, for the coordinates A(2,3) and B(6,6) we can find $\overrightarrow{AB}$ two different ways:

1. Go directly from A to B which is $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

2. Use the position vectos, so go from A to O and O to B

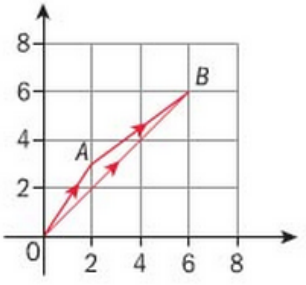
$\overrightarrow{AB}$ is the resultant vector

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}.$$

Recall that $\overrightarrow{AO} = -\overrightarrow{OA}$,

$$\overrightarrow{AB} = -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= \overrightarrow{OB} - \overrightarrow{OA}$$



→ To find the **resultant vector** $\overrightarrow{AB}$ between two points A and B we can subtract the position vector of A from the position vector of B.

Points A and B have coordinates $(-3, 2, 0)$ and $(-4, 7, 5)$.
Find the vector $\overrightarrow{AB}$.

If A is $(3, -1, 2)$ and B is $(1, 0, -2)$ find: **a** $\overrightarrow{OA}$ **b** $\overrightarrow{AB}$

3 **a** Given $\overrightarrow{BA} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ find $\overrightarrow{AC}$.

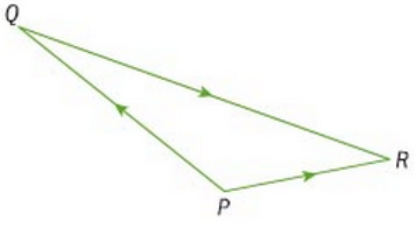
b If $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\overrightarrow{CA} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, find $\overrightarrow{CB}$.

c If $\overrightarrow{PQ} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, $\overrightarrow{RQ} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{RS} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$, find $\overrightarrow{SP}$.

Given that $\overrightarrow{XY} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ and $\overrightarrow{XZ} = \begin{pmatrix} 0 \\ -10 \\ -1 \end{pmatrix}$

Find the vectors **a** $\overrightarrow{YZ}$ **b** $\overrightarrow{ZY}$

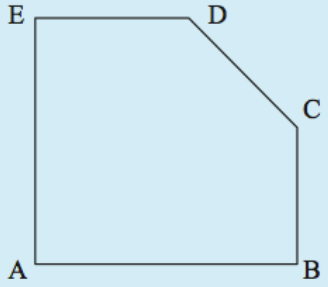
Similarly, this works if we don't have position vectors or can't write the position vectors.



Say we know $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

How would we write $\overrightarrow{QR}$?

Find the resultant vector for the following:



a $\overrightarrow{BC} + \overrightarrow{CA}$

b $\overrightarrow{BA} + \overrightarrow{AE} + \overrightarrow{EC}$

c $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$

d $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DE}$

By finding the resultant vector, we can find the distance between any two points.

*NOTE: You can use the distance formula here too!

If P is $(-3, 1, 2)$ and Q is $(1, -1, 3)$, find $|\overrightarrow{PQ}|$.

If A is $(-1, 3, 2)$ and B is $(2, 1, -4)$, find:

a the position vector of A from B

b the distance between A and B.

Given $M(4, -2, -1)$ and $N(-1, 2, 0)$ find:

a the position vector of M from N

b the position vector of N from M

c the distance between M and N.

For $A(-1, 2, 5)$, $B(2, 0, 3)$ and $C(-3, 1, 0)$ find the position vector of:

a A from O and the distance from O to A

b C from A and the distance from A to C

c B from C and the distance from C to B.

Application with Vectors

ABCD is a parallelogram. A is $(-1, 2, 1)$, B is $(2, 0, -1)$ and D is $(3, 1, 4)$. Find the coordinates of C.

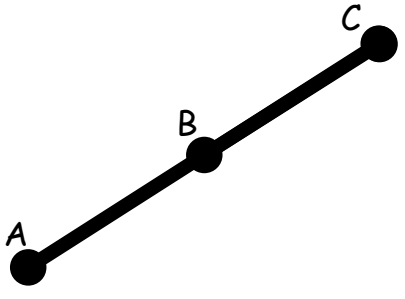
A quadrilateral has vertices $A(1, 2, 3)$, $B(3, -3, 2)$, $C(7, -4, 5)$ and $D(5, 1, 6)$.

a Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

b What can be deduced about the quadrilateral ABCD?

Collinear Points

Collinear points all lie in a straight line.



What are two important ideas that are true about AB and AC that we could use to determine if points A, B and C are collinear?

1.

2.

Now, extend this to vectors!

Show that the points A, B and C with position vectors $\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $-2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ and $4\mathbf{i} - 7\mathbf{j} + 7\mathbf{k}$ respectively are collinear.

4 The position vectors of A, B and C are given by $3\mathbf{i} + 4\mathbf{j}$, $x\mathbf{i}$, $\mathbf{i} - 2\mathbf{j}$ respectively. Find the value of x so that A, B and C are collinear and find the ratio $AB : BC$.

Find a and b if $J(-4, 1, 3)$, $K(2, -2, 0)$ and $L(a, b, 2)$ are collinear.

Hint: $\overrightarrow{JK}$ is parallel to $\overrightarrow{KL}$.

Homework

Chapter 12.1

12C: 1-6

12D: 1-3

12E: 1-4