

Lots of Vector Applications Practice!

1 An ocean liner is at $(6, -6)$, cruising at 10 km h^{-1} in the direction $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

A fishing boat is anchored at $(0, 0)$. Distances are in kilometres.

- a Find, in terms of \mathbf{i} and \mathbf{j} , the original position vector of the liner from the fishing boat.
- b Write an expression for the position vector of the liner at any time t hours after it has sailed from $(6, -6)$.
- c Find when the liner is due east of the fishing boat.
- d Find the time and position of the liner when it is nearest to the fishing boat.

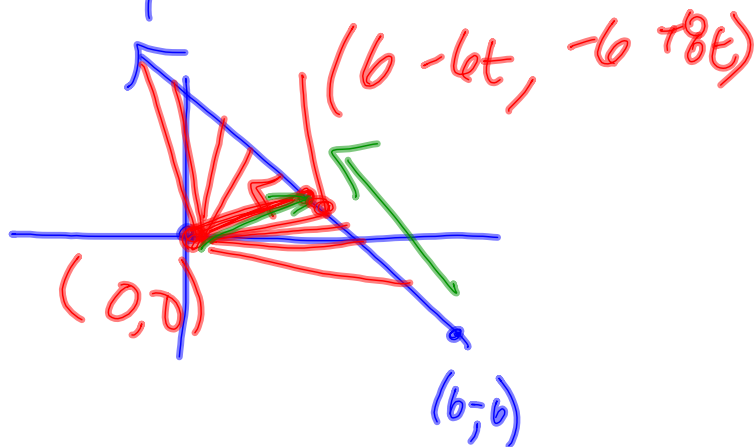
a) $6\mathbf{i} - 6\mathbf{j}$

b) $\begin{pmatrix} 6-6t \\ -6+8t \end{pmatrix}$

c) $0 = -6 + 8t$

$t = \frac{3}{4} = 0.75 \text{ hr}$

d)



$$\begin{pmatrix} 6-6t \\ -6+8t \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = 0$$

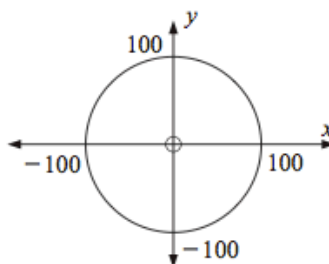
$t = \frac{21}{25} = \boxed{0.84 \text{ hr}}$ $(0.96, 0.72)$

Find velocity vector

- 2 Let $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represent a 1 km displacement due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent a 1 km displacement due north.

The control tower of an airport is at $(0, 0)$. Aircraft within 100 km of $(0, 0)$ will become visible on the radar screen at the control tower.

At 12:00 noon an aircraft is 200 km east and 100 km north of the control tower. It is flying parallel to the vector $\mathbf{b} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ with a speed of $40\sqrt{10} \text{ km h}^{-1}$.



- a Write down the velocity vector of the aircraft.
- b Write a vector equation for the path of the aircraft using t to represent the time in hours that have elapsed since 12:00 noon.
- c Find the position of the aircraft at 1:00 pm.
- d Show that the aircraft first becomes visible on the radar screen at 1:00 pm.
- e Find the time when the aircraft is closest to the control tower and find the distance between the aircraft and the control tower at this time.
- f At what time will the aircraft disappear from the radar screen?

4 Boat A's position is given by
 $x(t) = 3 - t$, $y(t) = 2t - 4$ where the distance
 units are kilometres and the time units are hours.
 Boat B's position is given by
 $x(t) = 4 - 3t$, $y(t) = 3 - 2t$.

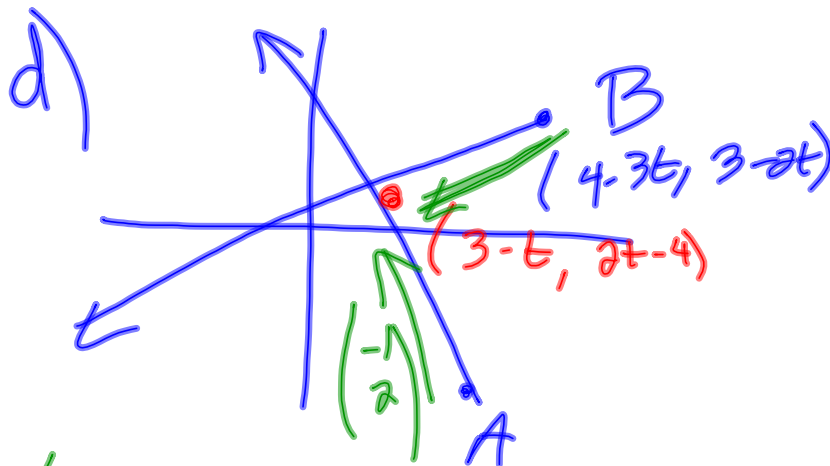
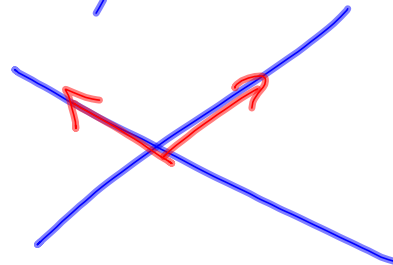


- a Find the initial position of each boat.
- b Find the velocity vector of each boat.
- c What is the angle between the paths of the boats?
- d At what time are the boats closest to each other?

a) $A: (3, -4)$ $B: (4, 3)$

b) $A: \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $B: \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

c) $\theta = 97.1^\circ$



$$\begin{pmatrix} 3-t - (4-3t) \\ 2t-4 - (3-2t) \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \end{pmatrix} = 0$$

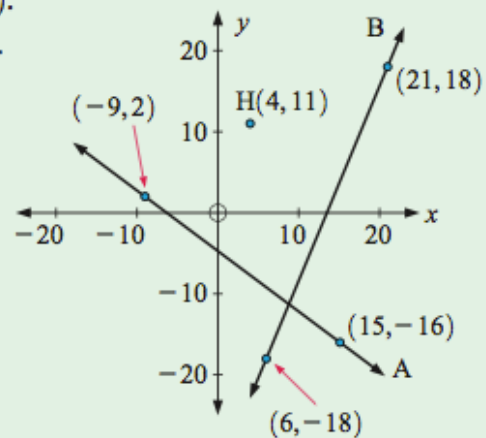
$$t = \frac{13}{6}$$

8 Let $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represent a 1 km displacement due east and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represent a 1 km displacement due north.

Road A passes through $(-9, 2)$ and $(15, -16)$.

Road B passes through $(6, -18)$ and $(21, 18)$.

- Find vector equations for each of the roads.
- An injured hiker is at $(4, 11)$, and needs to get to a road in the shortest possible distance. Towards which road should he head, and how far will he need to walk to reach the road?



- 7** Submarine X23 is at $(2, 4)$. It fires a torpedo with velocity vector $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ at exactly 2:17 pm. Submarine Y18 is at $(11, 3)$. It fires a torpedo with velocity vector $\begin{pmatrix} -1 \\ a \end{pmatrix}$ at 2:19 pm to intercept the torpedo from X23.
- Find the vector equation for the torpedo fired from submarine X23.
 - Find the vector equation for the torpedo fired from submarine Y18.
 - At what time does the interception occur?
 - What was the direction and speed of the interception torpedo?

EXAM-STYLE QUESTION

8 (In this question distances are measured in km and time in hours.) At noon a lighthouse keeper observes two ships A and B.

Ship A's position at time t is given by $\mathbf{r}_1 = \begin{pmatrix} -4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 17 \end{pmatrix}$.

Ship B's position at time t is given by $\mathbf{r}_2 = \begin{pmatrix} 4 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -12 \\ 5 \end{pmatrix}$.

a Show that A and B will collide, and find the time when this will occur and the position vector of the point of collision.

In order to prevent collision, at 12:15 ship A changes its direction to $\begin{pmatrix} 16 \\ 17 \end{pmatrix}$.

b Find the distance between A and B at 12:30.

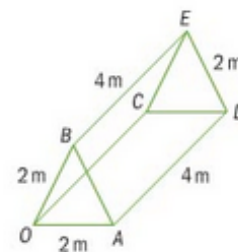
a) $t = 0.5$

12:30

$$\begin{pmatrix} -2 \\ 11.5 \end{pmatrix}$$

b) 6 km

- 3** A tent $OABCDE$ is a triangular prism with a constant cross-section that is an equilateral triangle with sides of 2 m. The tent is 4 m long. The base $OADC$ is horizontal. Support poles are to be laid along the diagonals BC and BD .



Take O as the origin and unit vectors \mathbf{i} and \mathbf{j} in the directions of OA and OC respectively, \mathbf{k} is a unit vector vertically upwards.

- a** Express these vectors in terms of the unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} .
- i** \vec{OC} **ii** \vec{OB} **iii** \vec{OD}
- b** Hence find vectors \vec{BC} and \vec{BD} .
- c** Calculate the values of
- i** $|\vec{BC}|$ **ii** $|\vec{BD}|$
- iii** the scalar product of \vec{BC} and \vec{BD} .
- d** Hence find the angle between the support poles.

- 6** All distances in this question are in metres and time is in seconds.

An insect is flying at a constant height. At time $t = 0$, the insect is at point A with coordinates $(0, 0, 6)$. Two seconds later the insect is at point B with coordinates $(6, -2, 6)$.

- a** Find vector \overrightarrow{AB} .

The insect continues to fly in the same direction at the same speed.

- b** Show that the position vector of the insect at time t is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + t \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}.$$

At time $t = 0$, a bird takes off from the ground. The position

vector of the bird at time t is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 36 \\ 18 \\ 0 \end{pmatrix} + t \begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$.

- c** Write down the coordinates of the starting position of the bird.
- d** Find the speed of the bird.
- The bird reaches the insect at point C .
- e** Find the time the bird takes to reach the insect.
- f** Find the coordinates of C .

Homework
Chapter 12.5
12L: All