# Lots of Vector Applications Practice!

**1** An ocean liner is at $(6, -6)$, cruising at $10 \text{ km h}^{-1}$ in the direction $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$.

velocity vector

A fishing boat is anchored at $(0, 0)$. Distances are in kilometres.

**a** Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the original position vector of the liner from the fishing boat.

**b** Write an expression for the position vector of the liner at any time $t$ hours after it has sailed from $(6, -6)$.

**c** Find when the liner is due east of the fishing boat.

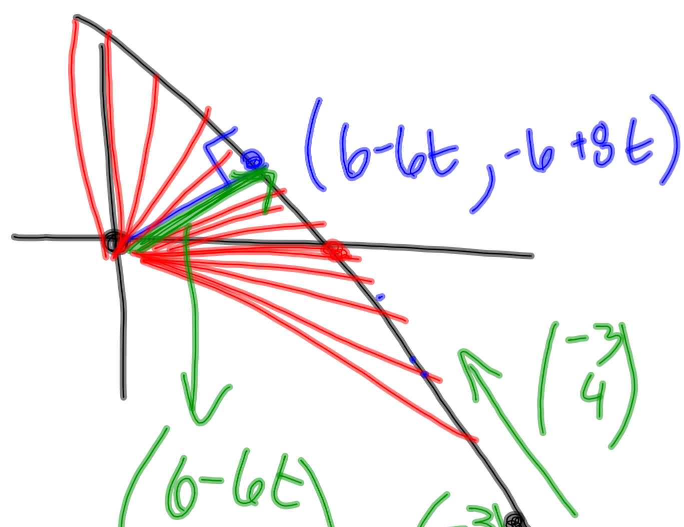
**d** Find the time and position of the liner when it is nearest to the fishing boat.

a) $6\vec{i} - 6\vec{j}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ -6 \end{pmatrix} + t\begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

b) $\begin{pmatrix} 6-6t \\ -6+8t \end{pmatrix}$

c) $t = \frac{3}{4}$ hr




$$\begin{pmatrix} 6-6t \\ -6+8t \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = 0$$

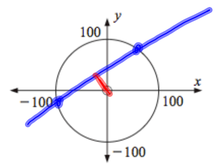
$t = 0.84$

$(0.96, 0.72)$

**2** Let $\begin{pmatrix}1\\0\end{pmatrix}$ represent a 1 km displacement due east and $\begin{pmatrix}0\\1\end{pmatrix}$ represent a 1 km displacement due north.

The control tower of an airport is at (0, 0). Aircraft within 100 km of (0, 0) will become visible on the radar screen at the control tower.

At 12:00 noon an aircraft is 200 km east and 100 km north of the control tower. It is flying parallel to the vector $\mathbf{b} = \begin{pmatrix}-3\\-1\end{pmatrix}$ with a speed of $40\sqrt{10}$ km h$^{-1}$.




**a** Write down the velocity vector of the aircraft.

**b** Write a vector equation for the path of the aircraft using $t$ to represent the time in hours that have elapsed since 12:00 noon.

**c** Find the position of the aircraft at 1:00 pm.

**d** Show that the aircraft first becomes visible on the radar screen at 1:00 pm.

**e** Find the time when the aircraft is closest to the control tower and find the distance between the aircraft and the control tower at this time.

**f** At what time will the aircraft disappear from the radar screen?

a) $\begin{pmatrix}-120\\-40\end{pmatrix}$

b) $\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}200\\100\end{pmatrix} + t\begin{pmatrix}-120\\-40\end{pmatrix}$

c) $(80, 60)$

d) enters radar at 100 km

$\left|\begin{pmatrix}80\\60\end{pmatrix}\right| = \sqrt{80^2+60^2} = 100$

e) $\begin{pmatrix}200-120t\\100-40t\end{pmatrix} \cdot \begin{pmatrix}-3\\-1\end{pmatrix} = 0$

$\vdots$

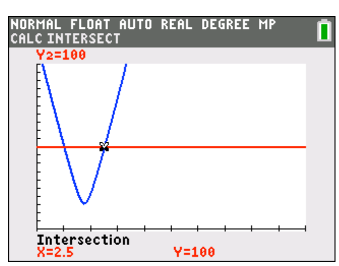
$t = 1.75$ 1:45 pm

$\left|\begin{pmatrix}-10\\30\end{pmatrix}\right| \approx 31.6$ km

f) $\left|\begin{pmatrix}200-120t\\100-40t\end{pmatrix}\right| = 100$

$\sqrt{(200-120t)^2 + (100-40t)^2} = 100$

$\boxed{t = 2.5}$



4 Boat A's position is given by $x(t) = 3 - t,\ \ y(t) = 2t - 4$ where the distance units are kilometres and the time units are hours. Boat B's position is given by $x(t) = 4 - 3t,\ \ y(t) = 3 - 2t$.

a Find the initial position of each boat.

b Find the velocity vector of each boat.

c What is the angle between the paths of the boats?

d At what time are the boats closest to each other?

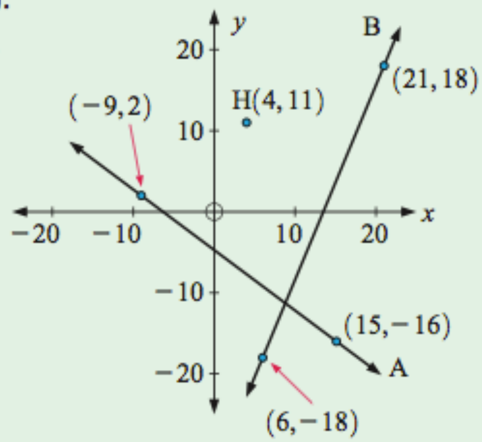
**8** Let $\begin{pmatrix}1\\0\end{pmatrix}$ represent a 1 km displacement due east and $\begin{pmatrix}0\\1\end{pmatrix}$ represent a 1 km displacement due north.

Road A passes through $(-9, 2)$ and $(15, -16)$.

Road B passes through $(6, -18)$ and $(21, 18)$.

**a** Find vector equations for each of the roads.

**b** An injured hiker is at $(4, 11)$, and needs to get to a road in the shortest possible distance. Towards which road should he head, and how far will he need to walk to reach the road?



**7** Submarine X23 is at (2, 4). It fires a torpedo with velocity vector $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ at exactly 2:17 pm. Submarine Y18 is at (11, 3). It fires a torpedo with velocity vector $\begin{pmatrix} -1 \\ a \end{pmatrix}$ at 2:19 pm to intercept the torpedo from X23.

- **a** Find the vector equation for the torpedo fired from submarine X23.
- **b** Find the vector equation for the torpedo fired from submarine Y18.
- **c** At what time does the interception occur?
- **d** What was the direction and speed of the interception torpedo?

EXAM-STYLE QUESTION

**8** (In this question distances are measured in km and time in hours.) At noon a lighthouse keeper observes two ships A and B.

Ship A's position at time $t$ is given by $\mathbf{r}_1 = \begin{pmatrix}-4\\3\end{pmatrix} + \lambda\begin{pmatrix}4\\17\end{pmatrix}$.

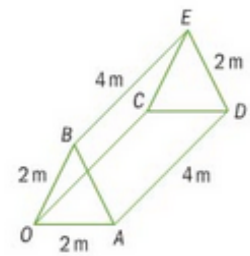
Ship B's position at time $t$ is given by $\mathbf{r}_2 = \begin{pmatrix}4\\9\end{pmatrix} + \mu\begin{pmatrix}-12\\5\end{pmatrix}$.

**a** Show that A and B will collide, and find the time when this will occur and the position vector of the point of collision.

In order to prevent collision, at 12:15 ship A changes its direction to $\begin{pmatrix}16\\17\end{pmatrix}$.

**b** Find the distance between A and B at 12:30.

3 A tent *OABCDE* is a triangular prism with a constant cross-section that is an equilateral triangle with sides of 2 m. The tent is 4 m long. The base *OADC* is horizontal. Support poles are to be laid along the diagonals *BC* and *BD*.




Take *O* as the origin and unit vectors **i** and **j** in the directions of *OA* and *OC* respectively, **k** is a unit vector vertically upwards.

**a** Express these vectors in terms of the unit vectors **i**, **j** and **k**.

**i** $\overrightarrow{OC}$ **ii** $\overrightarrow{OB}$ **iii** $\overrightarrow{OD}$

**b** Hence find vectors $\overrightarrow{BC}$ and $\overrightarrow{BD}$.

**c** Calculate the values of

**i** $|\overrightarrow{BC}|$ **ii** $|\overrightarrow{BD}|$

**iii** the scalar product of $\overrightarrow{BC}$ and $\overrightarrow{BD}$.

**d** Hence find the angle between the support poles.

**6** All distances in this question are in metres and time is in seconds.

An insect is flying at a constant height. At time $t = 0$, the insect is at point $A$ with coordinates (0, 0, 6). Two seconds later the insect is at point $B$ with coordinates (6, −2, 6).

**a** Find vector $\overrightarrow{AB}$.

The insect continues to fly in the same direction at the same speed.

**b** Show that the position vector of the insect at time $t$ is given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + t\begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}.$$

At time $t = 0$, a bird takes off from the ground. The position vector of the bird at time $t$ is given by $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 36 \\ 18 \\ 0 \end{pmatrix} + t\begin{pmatrix} -3 \\ -4 \\ 1 \end{pmatrix}$.

**c** Write down the coordinates of the starting position of the bird.

**d** Find the speed of the bird.

The bird reaches the insect at point $C$.

**e** Find the time the bird takes to reach the insect.

**f** Find the coordinates of $C$.

Homework

Chapter 12.5

12L: All