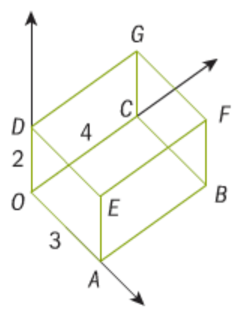
Vectors Skills Check - things you'll need to know for this unit

1 The cuboid, $OABCDEFG$ is such that OA has length 3 units, OC 4 units and OD 2 units. A lies on the x-axis, C on the y-axis and D on the z-axis.



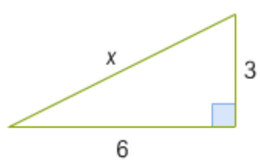
Give the coordinates of

a A **b** B

c E **d** F

e H, the midpoint of GF.

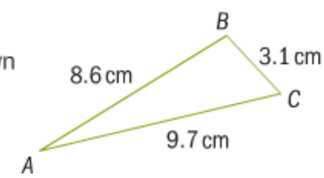
2 Find the length of the hypotenuse, x.



3 a In triangle ABC, $AB = 9$ cm, $BC = 15$ cm and angle $ABC = 110°$. Calculate the length of AC. correct to the nearest cm.

b In triangle ABC, $AB = 8.6$ cm, $BC = 3.1$cm and $AC = 9.7$cm.

Diagram NOT accurately drawn



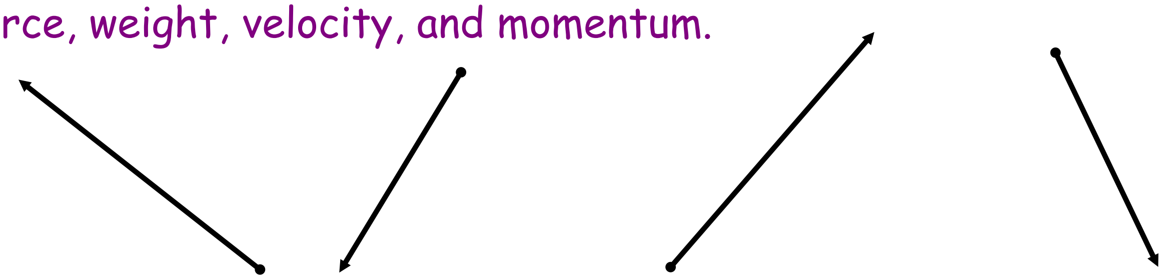
Calculate angle ABC to the nearest degree.

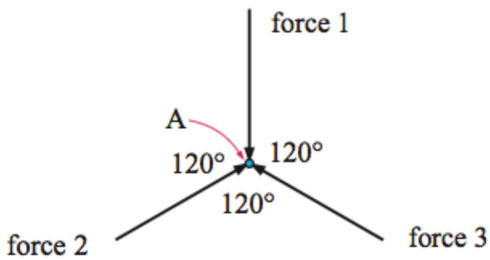
Introduction to Vectors!

Why do we need vectors? What do they tell us?

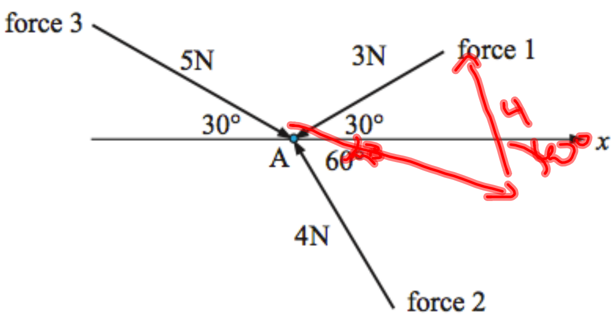
If I tell you we are 340km from Paris, what important information is missing?

Vectors are extensively used in Physics to represent displacement, force, weight, velocity, and momentum.





If 3 equal forces are acting on an object all at 120 , will it move?



What if the forces act on the object in different directions?

What is the resultant force?

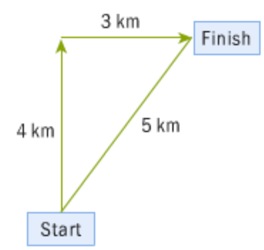
What direction will the object move?

If you travel 4 kilometres north and 3 kilometres east, how far have you traveled?

Maybe a very simple question... BUT we have two different ways to answer!

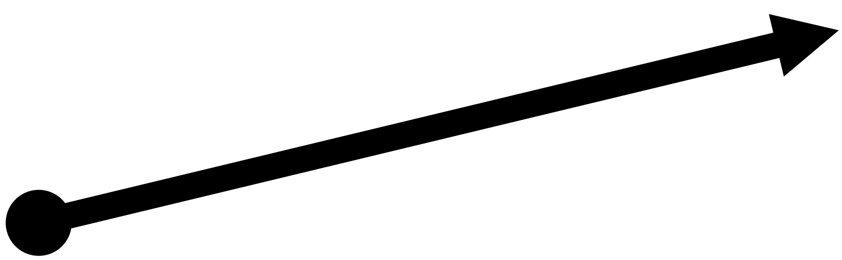
- One answer to this question is to say that you have traveled 7 kilometres. This is the total **distance** that you have moved through (4 + 3 = 7 kilometres).

- A second answer to this question is to say that you have traveled 5 kilometres. This value has been found using Pythagoras' theorem ($\sqrt{4^2+3^2}$ = 5 kilometres). This value is called the **displacement**. It measures the difference between your initial position and your final position.



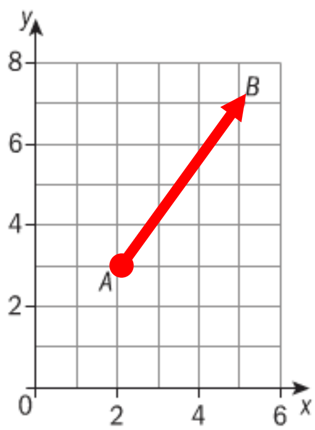
What is a vector?

- A **vector** is a quantity that has **size** (magnitude) and **direction**. Examples of vectors are displacement and velocity.
- A **scalar** is a quantity that has size but no direction. Examples of scalars are distance and speed.



Representation of vectors

Consider the points $A(2,3)$ and $B(5,7)$ on the Cartesian plane:



In the diagram the line AB represents the vector $\overrightarrow{AB}$ where the arrow over the letters indicates the direction of the movement (from A to B). The components of the vector are here represented using **column vector form.**

$$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Vectors can also be represented using a lower case **bold** letter. For example we could use **a** to represent the vector $\overrightarrow{AB}$.

$$\mathbf{a} = \overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

In a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$, the x represents a movement in the positive x direction and the y a movement in the positive y direction.

Bold letters are difficult to write by hand so instead when writing you should underline the letter to show that it is a vector.
So **a** is written by hand as $\underline{a}$.

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

We have two ways to write vectors:

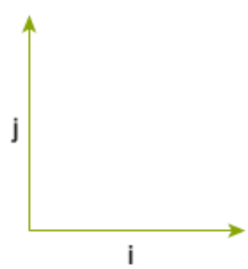
Column Vector Form

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

Unit Vector Form

$3\mathbf{i} + 4\mathbf{j}$ where $\mathbf{i}$ and $\mathbf{j}$ are vectors of length 1 unit

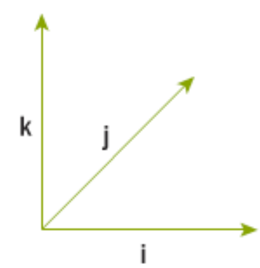
$\mathbf{i}$ and $\mathbf{j}$ are called base vectors.



3D Vectors

$$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$$

k is the 3rd unit vector in 3D



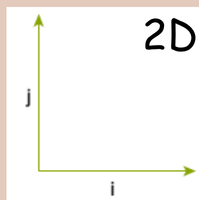
→ The unit vector in the direction of the *x*-axis is **i**.

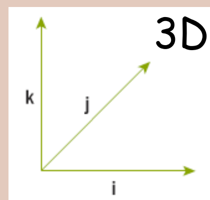
In two dimensions $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and in three dimensions $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

→ The unit vector in the direction of the *y*-axis is **j**. In two dimensions $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and in three dimensions $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

→ In three dimensions the unit vector in the direction of the *z*-axis is

$$\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$





The vectors **i**, **j**, **k** are called **base vectors**.

i -> x direction; j -> y direction; k -> z direction

a Write $\mathbf{a} = \begin{pmatrix} 6 \\ -7 \end{pmatrix}$ in unit vector form.

b Write $-\mathbf{i} + 5\mathbf{k}$ in column vector form.

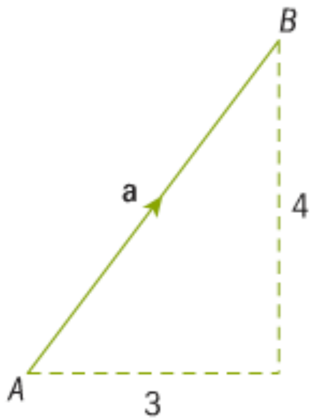
a) $6i - 7j = \underline{a}$

b) $\begin{pmatrix} -1 \\ 0 \\ 5 \end{pmatrix}$

The magnitude of a vector

The **magnitude** of $\overrightarrow{AB}$ is the length of the vector and is denoted by $|\overrightarrow{AB}|$.
Magnitude is found by using Pythagoras' theorem.

Other names for magnitude are modulus, length, norm and size.



If $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ then $|\overrightarrow{AB}| = 5$

Magnitude in 2D

→ If $\overrightarrow{AB} = \begin{pmatrix} a \\ b \end{pmatrix} = a\mathbf{i} + b\mathbf{j}$ then $|\overrightarrow{AB}| = \sqrt{a^2 + b^2}$.

Magnitude in 3D

→ If $\overrightarrow{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ then $|\overrightarrow{AB}| = \sqrt{a^2 + b^2 + c^2}$

What is looks like in your booklet...

4.1	Magnitude of a vector	$\lvert \boldsymbol{v} \rvert = \sqrt{v_1^{\ 2} + v_2^{\ 2} + v_3^{\ 2}}$

Find the magnitude of these vectors

a $\overrightarrow{OP} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$ **b** $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$

a) $|\overrightarrow{OP}| = 13$

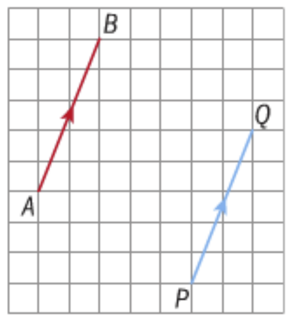
b) $\left|\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\right| = \sqrt{14}$

When physicists deal with problems of 'uniform acceleration' and 'free fall under gravity' they need to consider the magnitude and direction of the acceleration vector. You may wish to explore this concept further.

Equal, negative and parallel vectors

→ Two vectors are **equal** if they have the same direction and the same magnitude; their **i**, **j**, **k** components are equal too, and so their column vectors are equal.

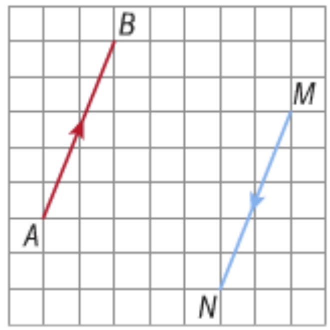
Vectors $\overrightarrow{AB}$ and $\overrightarrow{PQ}$ are pointing in the same direction (are parallel) and have the same magnitude. Therefore $\overrightarrow{AB} = \overrightarrow{PQ}$.



It does not matter where in the Cartesian plane these vectors are – they are still equal.

If two vectors are equal in length then their components will be the same.
Here $\overrightarrow{AB} = \overrightarrow{PQ} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

The two vectors $\overrightarrow{AB}$ and $\overrightarrow{MN}$ have the same magnitude but different directions.
So $\overrightarrow{AB} \neq \overrightarrow{MN}$.

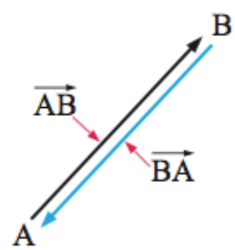


The direction of a vector is important, not just its length.

Here $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{MN} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ and so $\overrightarrow{AB} = -\overrightarrow{MN}$.

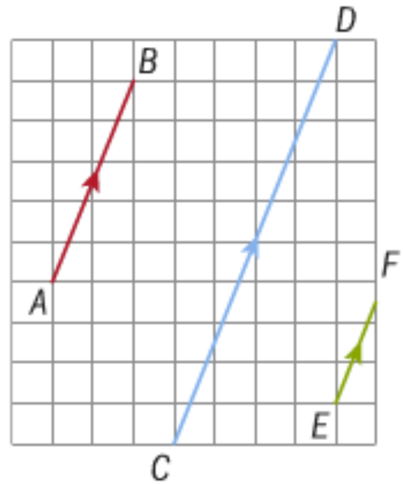
$\overrightarrow{MN}$ is called the **negative vector**.

→ You can write $\overrightarrow{AB}$ as $-\overrightarrow{BA}$.



Vectors $\overrightarrow{AB}$, $\overrightarrow{CD}$ and $\overrightarrow{EF}$ are all **parallel** but have different magnitudes.

Here $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{CD}$ and $\overrightarrow{AB} = 2\overrightarrow{EF}$.



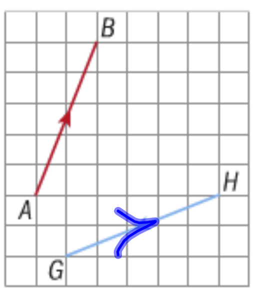
$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 2\mathbf{i} + 5\mathbf{j}$

$\overrightarrow{CD} = \begin{pmatrix} 4 \\ 10 \end{pmatrix} = 4\mathbf{i} + 10\mathbf{j}$

$\overrightarrow{EF} = \begin{pmatrix} 1 \\ 2.5 \end{pmatrix} = 1\mathbf{i} + 2.5\mathbf{j}$

→ Two vectors are **parallel** if one is a scalar multiple of the other. So, $\overrightarrow{AB}$ and $\overrightarrow{RS}$ are parallel if $\overrightarrow{AB} = k\overrightarrow{RS}$ where k is a scalar quantity. This can also be written as $\mathbf{a} = k\mathbf{b}$.

Vectors $\overrightarrow{AB}$ and $\overrightarrow{GH}$ both have a magnitude of 29 but different directions. So $\overrightarrow{AB} \neq \overrightarrow{GH}$ and they are not parallel

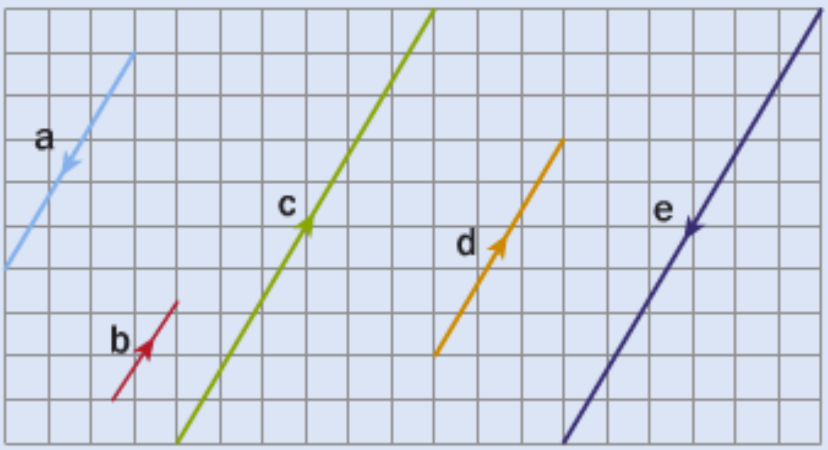


$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 5 \end{pmatrix} = 2\mathbf{i} + 5\mathbf{j}$

$\overrightarrow{GH} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} = 5\mathbf{i} + 2\mathbf{j}$

We cannot multiply $\overrightarrow{AB}$ by a scalar to get $\overrightarrow{GH}$.

The diagram shows several vectors.



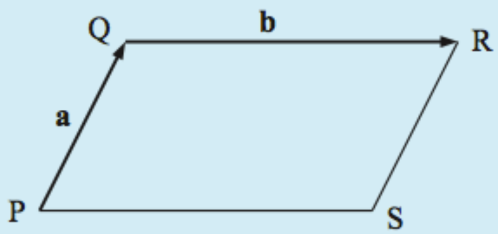
Write each of the other vectors in terms of the vector **a**.

$\underline{b} = -\frac{1}{2}\underline{a}$

$\underline{d} = -\underline{a}$

$\underline{c} = -2\underline{a}$

$\underline{e} = 2\underline{a}$



PQRS is a parallelogram in which $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{QR} = \mathbf{b}$.

Find vector expressions for:

a $\overrightarrow{QP}$ **b** $\overrightarrow{RQ}$ **c** $\overrightarrow{SR}$ **d** $\overrightarrow{SP}$

a) $-\underline{a}$ c) $\underline{a}$

b) $-\underline{b}$ d) $-\underline{b}$

For what values of t and s are these two vectors parallel?
$\mathbf{m} = 3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ and $\mathbf{n} = 9\mathbf{i} - 12\mathbf{j} + s\mathbf{k}$

$t = -4$

$s = -18$

$\underline{m} \cdot k = \underline{n}$

$\underline{m} = k \cdot \underline{n}$ X

$$k\begin{pmatrix}3\\t\\-6\end{pmatrix} = \begin{pmatrix}9\\-12\\s\end{pmatrix}$$

$3k = 9 \rightarrow k = 3$

$kt = -12$

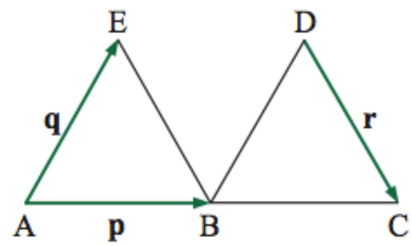
$-6k = s$

The figure alongside consists of 2 equilateral triangles. A, B and C lie on a straight line.
$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AE} = \mathbf{q}$ and $\overrightarrow{DC} = \mathbf{r}$.

Which of the following statements are true?

a $\overrightarrow{EB} = \mathbf{r}$ b $|\mathbf{p}| = |\mathbf{q}|$ c $\overrightarrow{BC} = \mathbf{r}$

d $\overrightarrow{DB} = \mathbf{q}$ e $\overrightarrow{ED} = \mathbf{p}$ f $\mathbf{p} = \mathbf{q}$



Homework

Chapter 12

12A & 12B