Vectors Review Paper 1

1a. *[2 marks]*

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

Write down a vector equation for line L.

1b. *[6 marks]*

The line L intersects the x-axis at the point P. Find the x-coordinate of P.

2a. *[2 marks]*

Consider points A$(1, -2, -1)$, B$(7, -4, 3)$ and C$(1, -2, 3)$. The line L_1 passes through C and is parallel to $\overrightarrow{\mathrm{AB}}$.

Find $\overrightarrow{\mathrm{AB}}$.

2b. *[2 marks]*

Hence, write down a vector equation for L_1.

2c. *[3 marks]*

A second line, L_2, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s\begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

Given that L_1 is perpendicular to L_2, show that $p = -6$.

2d. *[7 marks]*

The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q.

3a. *[1 mark]*

The line L_1 passes through the points $\mathrm{A}(2, 1, 4)$ and $\mathrm{B}(1, 1, 5)$.

Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

3b. *[1 mark]*

Hence, write down a direction vector for L_1;

3c. *[2 marks]*

Hence, write down a vector equation for L_1.

3d. *[6 marks]*

Another line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

Find the coordinates of P.

3e. *[1 mark]*

Write down a direction vector for L_2.

3f. *[6 marks]*

Hence, find the angle between L_1 and L_2.

4a. *[2 marks]*

The line L is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Find the gradient of the line L.

4b. *[3 marks]*

The line L passes through the point $(9, 4)$.

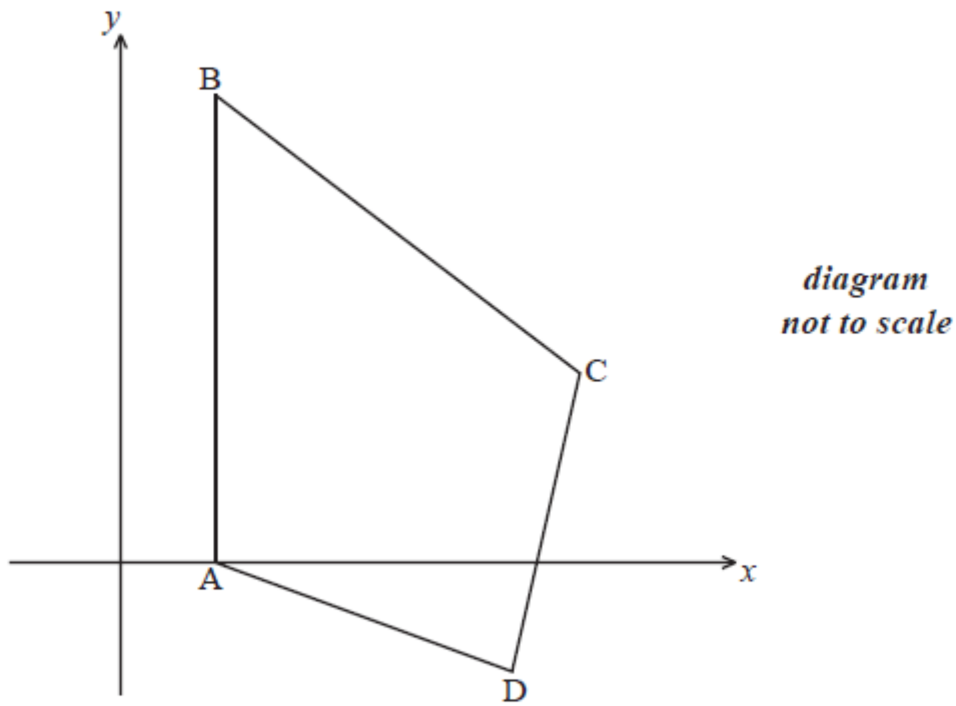
Find the equation of the line L in the form $y = ax + b$.

4c. *[2 marks]*

Write down a vector equation for the line L.

5a. *[5 marks]*

The diagram shows quadrilateral ABCD with vertices A(1, 0), B(1, 5), C(5, 2) and D(4, −1) .



(i) Show that $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(ii) Find $\overrightarrow{BD}$.

(iii) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$.

5b. *[4 marks]*

The line (AC) has equation $\boldsymbol{r} = \boldsymbol{u} + s\boldsymbol{v}$.

(i) Write down vector $\boldsymbol{u}$ and vector $\boldsymbol{v}$.

(ii) Find a vector equation for the line (BD).

5c. *[3 marks]*

The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\, k)$.

Show that $k = 1$.

5d. *[5 marks]*

The lines (AC) and (BD) intersect at the point $\mathrm{P}(3,\, k)$.

Hence find the area of triangle ACD.

6a. *[2 marks]*

The line L_1 is represented by the vector equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ -1 \\ -25 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -8 \end{pmatrix}$.

A second line L_2 is parallel to L_1 and passes through the point B$(-8, -5, 25)$.

Write down a vector equation for L_2 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

6b. *[5 marks]*

A third line L_3 is perpendicular to L_1 and is represented by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 3 \end{pmatrix} + q \begin{pmatrix} -7 \\ -2 \\ k \end{pmatrix}$.

Show that $k = -2$.

6c. *[6 marks]*

The lines L_1 and L_3 intersect at the point A.

Find the coordinates of A.

6d. *[5 marks]*

The lines L_2 and L_3 intersect at point C where $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 6 \\ 3 \\ -24 \end{pmatrix}$.

(i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, find $|\overrightarrow{\mathrm{AC}}|$.

7a. *[2 marks]*

Let $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ -3 \\ 2 \end{pmatrix}$.

Find $\overrightarrow{\mathrm{BC}}$.

7b. *[3 marks]*

Find a unit vector in the direction of $\overrightarrow{\mathrm{AB}}$.

7c. *[3 marks]*

Show that $\overrightarrow{\mathrm{AB}}$ is perpendicular to $\overrightarrow{\mathrm{AC}}$.

8a. *[3 marks]*

Let $\boldsymbol{u} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}$. Given that $\boldsymbol{u}$ is perpendicular to $\boldsymbol{w}$, find the value of p.

8b. *[3 marks]*

Let $\boldsymbol{v} = \begin{pmatrix} 1 \\ q \\ 5 \end{pmatrix}$. Given that $|\boldsymbol{v}| = \sqrt{42}$, find the possible values of q.

9a. *[3 marks]*

The vertices of the triangle PQR are defined by the position vectors

$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\overrightarrow{\mathrm{OR}} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}$.

Find

(i) $\overrightarrow{\mathrm{PQ}}$;

(ii) $\overrightarrow{\mathrm{PR}}$.

9b. *[7 marks]*

Show that $\cos \mathrm{R\hat{P}Q} = \frac{1}{2}$.

9c. *[6 marks]*

(i) Find $\sin \mathrm{R\hat{P}Q}$.

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

10. *[6 marks]*

Find the cosine of the angle between the two vectors $3\boldsymbol{i} + 4\boldsymbol{j} + 5\boldsymbol{k}$ and $4\boldsymbol{i} - 5\boldsymbol{j} - 3\boldsymbol{k}$.

11a. *[2 marks]*

The line L_1 is parallel to the *z*-axis. The point P has position vector $\begin{pmatrix} 8 \\ 1 \\ 0 \end{pmatrix}$ and lies on L_1.

Write down the equation of L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

11b. *[4 marks]*

The line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix}$. The point A has position vector $\begin{pmatrix} 6 \\ 2 \\ 9 \end{pmatrix}$.

Show that A lies on L_2.

11c. *[7 marks]*

Let B be the point of intersection of lines L_1 and L_2 .

(i) Show that $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 8 \\ 1 \\ 14 \end{pmatrix}$.

(ii) Find $\overrightarrow{\mathrm{AB}}$.

11d. *[3 marks]*

The point C is at (2, 1, − 4). Let D be the point such that ABCD is a parallelogram.

Find $\overrightarrow{\mathrm{OD}}$.

12a. *[4 marks]*

The line L_1 passes through the points P(2, 4, 8) and Q(4, 5, 4) .

(i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$.

12b. *[7 marks]*

The line L_2 is perpendicular to L_1 , and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(i) Find the value of *p* .

(ii) Given that L_2 passes through $\mathrm{R}(10, 6, -40)$, write down a vector equation for L_2 .

12c. *[7 marks]*

The lines L_1 and L_2 intersect at the point A. Find the *x*-coordinate of A.

13a. *[2 marks]*

A line L passes through $\mathrm{A}(1,\ -1,\ 2)$ and is parallel to the line $\boldsymbol{r} = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix} + s\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$.

Write down a vector equation for L in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$.

13b. *[4 marks]*

The line L passes through point P when $t = 2$.

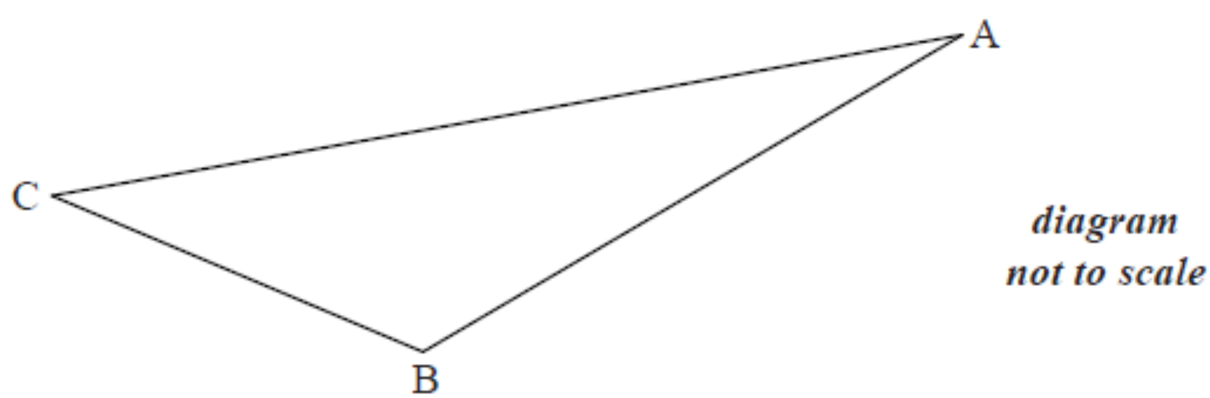
Find

(i) $\overrightarrow{\mathrm{OP}}$;

(ii) $|\overrightarrow{\mathrm{OP}}|$.

14a. *[3 marks]*

The following diagram shows the obtuse-angled triangle ABC such that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -3 \\ 0 \\ -4 \end{pmatrix}$ and $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -2 \\ 2 \\ -6 \end{pmatrix}$.



(i) Write down $\overrightarrow{\mathrm{BA}}$.

(ii) Find $\overrightarrow{\mathrm{BC}}$.

14b. *[7 marks]*

(i) Find $\cos A\hat{B}C$.

(ii) Hence, find $\sin A\hat{B}C$.

14c. *[6 marks]*

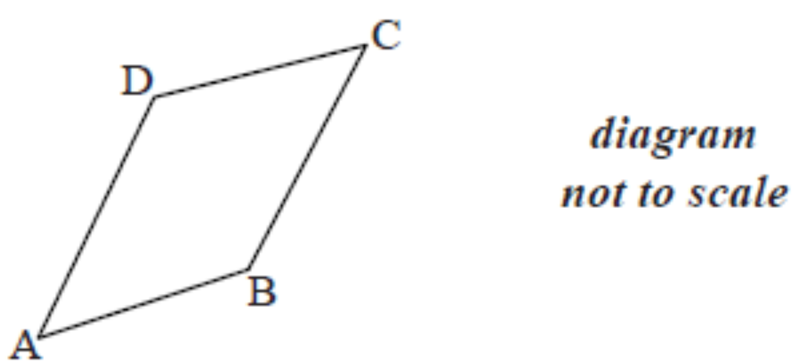
The point D is such that $\overrightarrow{CD} = \begin{pmatrix} -4 \\ 5 \\ p \end{pmatrix}$, where $p > 0$.

(i) Given that $|\overrightarrow{CD}| = \sqrt{50}$, show that $p = 3$.

(ii) Hence, show that $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{BC}$.

15a. *[2 marks]*

The following diagram shows quadrilateral ABCD, with $\overrightarrow{AD} = \overrightarrow{BC}$, $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$.



Find $\overrightarrow{BC}$.

15b. *[2 marks]*

Show that $\overrightarrow{BD} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$.

15c. *[3 marks]*

Show that vectors $\overrightarrow{BD}$ and $\overrightarrow{AC}$ are perpendicular.

16a. *[4 marks]*

A particle is moving with a constant velocity along line L . Its initial position is A(6 , −2 , 10) . After one second the particle has moved to B(9, −6 , 15) .

(i) Find the velocity vector, $\overrightarrow{\mathrm{AB}}$.

(ii) Find the speed of the particle.

16b. *[2 marks]*

Write down an equation of the line L .

17a. *[3 marks]*

Consider the points A (1 , 5 , 4) , B (3 , 1 , 2) and D (3 , k , 2) , with (AD) perpendicular to (AB) .

Find

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AD}}$ giving your answer in terms of k .

[3 marks]

17b. *[3 marks]*

Show that $k = 7$.

17c. *[4 marks]*

The point O has coordinates (0 , 0 , 0) , point A has coordinates (1 , – 2 , 3) and point B has coordinates (– 3 , 4 , 2) .

The point C is such that $\overrightarrow{\mathrm{BC}} = \frac{1}{2}\overrightarrow{\mathrm{AD}}$.

Find the position vector of C.

17d. *[3 marks]*

Find $\cos \mathrm{A\hat{B}C}$.