Trigonometry Test Review Paper 2 KEY

1. (a) evidence of choosing cosine rule (M1)

e.g. $a^2 + b^2 - 2ab\cos C$

correct substitution A1

e.g. $7^2 + 9^2 - 2(7)(9)\cos 120^\circ$

AC $= 13.9\ (= \sqrt{193})$ A1 N2 3

(b) **METHOD 1**

evidence of choosing sine rule (M1)

e.g. $\frac{\sin \hat{A}}{\text{BC}} = \frac{\sin \hat{B}}{\text{AC}}$

correct substitution A1

e.g. $\frac{\sin \hat{A}}{9} = \frac{\sin 120}{13.9}$

$\hat{A} = 34.1^\circ$ A1 N2 3

METHOD 2

evidence of choosing cosine rule (M1)

e.g. $\cos \hat{A} = \frac{\text{AB}^2 + \text{AC}^2 - \text{BC}^2}{2(\text{AB})(\text{AC})}$

correct substitution A1

e.g. $\cos \hat{A} = \frac{7^2 + 13.9^2 - 9^2}{2(7)(13.9)}$

$\hat{A} = 34.1^\circ$ A1 N2 3

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2. (a) valid approach (M1)

e.g. 15 mins is half way, top of the wheel, $d + 1$

height = 101 (metres) A1 N2

(b) evidence of identifying rotation angle after 6 minutes A1

e.g. $\frac{2\pi}{5}, \frac{1}{5}$ of a rotation, 72°

evidence of appropriate approach (M1)

e.g. drawing a right triangle and using cosine ratio

correct working (seen anywhere) A1

e.g. $\cos\frac{2\pi}{5} = \frac{x}{50}$, 15.4(508...)

evidence of appropriate method M1

e.g. height = radius + 1 – 15.45...

height 35.5 (metres) (accept 35.6) A1 N2

(c) **METHOD 1**

evidence of substituting into $b = \frac{2\pi}{\text{period}}$ (M1)

correct substitution

e.g. period = 30 minutes, $b = \frac{2\pi}{30}$ A1

$b = 0.209 \left(\frac{\pi}{15}\right)$ A1 N2

substituting into $h(t)$ (M1)

e.g. $h(0) = 1$, $h(15) = 101$

correct substitution A1

$1 = 50\sin\left(-\frac{\pi}{15}c\right) + 51$

$c = 7.5$ A1 N2

METHOD 2

evidence of setting up a system of equations (M1)

two correct equations

e.g. $1 = 50\sin b(0 - c) + 51$, $101 = 50\sin b(15 - c) + 51$ A1A1

attempt to solve simultaneously (M1)

e.g. evidence of combining two equations

$b = 0.209 \left(\frac{\pi}{15}\right)$, $c = 7.5$ A1A1 N2N2

(d) evidence of solving $h(t) = 96$ (M1)

e.g. equation, graph

$t = 12.8$ (minutes) A2 N3

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3. (a) **METHOD 1**

choosing cosine rule (M1)
substituting correctly A1

e.g. $AB = \sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$

AB = 6.11(cm) A1 N2

METHOD 2

evidence of approach involving right-angled triangles (M1)
substituting correctly A1

e.g. $\sin 0.9 = \frac{x}{3.9}, \frac{1}{2}AB = 3.9 \sin 0.9$

AB = 6.11 (cm) A1 N2

METHOD 3

choosing the sine rule (M1)
substituting correctly A1

e.g. $\frac{\sin 0.670...}{3.9} = \frac{\sin 1.8}{AB}$

AB = 6.11 (cm) A1 N2

(b) **METHOD 1**

reflex $A\hat{O}B = 2\pi - 1.8$ (= 4.4832) (A2)

correct substitution $A = \frac{1}{2}(3.9)^2(4.4832...)$ A1

area = 34.1 (cm^2) A1 N2

METHOD 2

finding area of circle $A = \pi(3.9)^2$ (= 47.78...) (A1)

finding area of (minor) sector $A = \frac{1}{2}(3.9)^2(1.8)$ (= 13.68...) (A1)

subtracting M1
e.g. $\pi(3.9)^2 - 0.5(3.9)^2(1.8)$, 47.8 – 13.7
area = 34.1 (cm^2) A1 N2

METHOD 3

finding reflex $A\hat{O}B = 2\pi - 1.8$ (= 4.4832) (A2)
finding proportion of total area of circle A1

e.g. $\frac{2\pi - 1.8}{2\pi} \times \pi(3.9)^2, \frac{\theta}{2\pi} \times \pi r^2$

area = 34.1 (cm) A1 N2

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Trigonometry Test Review Paper 2 KEY

4. (a) attempt to form any composition (even if order is reversed) (M1)

correct composition $h(x) = g\left(\frac{3x}{2}+1\right)$ (A1)

$h(x) = 4\cos\left(\frac{\frac{3x}{2}+1}{3}\right)-1 \quad \left(4\cos\left(\frac{1}{2}x+\frac{1}{3}\right)-1, 4\cos\left(\frac{3x+2}{6}\right)-1\right)$ A1 N3

(b) period is $4\pi (12.6)$ A1 N1

(c) range is $-5 \le h(x) \le 3$ ($[-5, 3]$) A1A1 N2

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5. (a) **METHOD 1**

evidence of choosing the cosine formula (M1)
correct substitution A1

e.g. $\cos A\hat{C}B = \frac{7^2+7^2-13^2}{2\times 7\times 7}$

$A\hat{C}B = 2.38$ radians ($= 136°$) A1 N2

METHOD 2

evidence of **appropriate** approach involving right-angled triangles (M1)
correct substitution A1

e.g. $\sin\left(\frac{1}{2}A\hat{C}B\right) = \frac{6.5}{7}$

$A\hat{C}B = 2.38$ radians ($= 136°$) A1 N2

(b) **METHOD 1**

$A\hat{C}D = \pi - 2.381$ ($180 - 136.4$) (A1)
evidence of choosing the sine rule in triangle ACD (M1)
correct substitution A1

e.g. $\frac{6.5}{\sin 0.760...} = \frac{7}{\sin A\hat{D}C}$

$A\hat{D}C = 0.836...$ ($= 47.9...°$) A1

$C\hat{A}D = \pi - (0.760... + 0.836...)$ $(180 - (43.5... + 47.9...))$
$= 1.54$ ($= 88.5°$) A1 N3

METHOD 2

$A\hat{B}C = \frac{1}{2}(\pi - 2.381)$ $\left(\frac{1}{2}(180 - 136.4)\right)$ (A1)

evidence of choosing the sine rule in triangle ABD (M1)

correct substitution A1

e.g. $\frac{6.5}{\sin 0.380...} = \frac{13}{\sin A\hat{D}C}$

$A\hat{D}C$ = 0.836... (= 47.9...°) A1

$C\hat{A}D = \pi - 0.836... - (\pi - 2.381...)$ (= 180 – 47.9... – (180 – 136.4))

= 1.54 (= 88.5°) A1 N3

Note: *Two triangles are possible with the given information. If candidate finds $A\hat{D}C$ = 2.31 (132°) leading to $C\hat{A}D$ = 0.076 (4.35°), award marks as per markscheme.*

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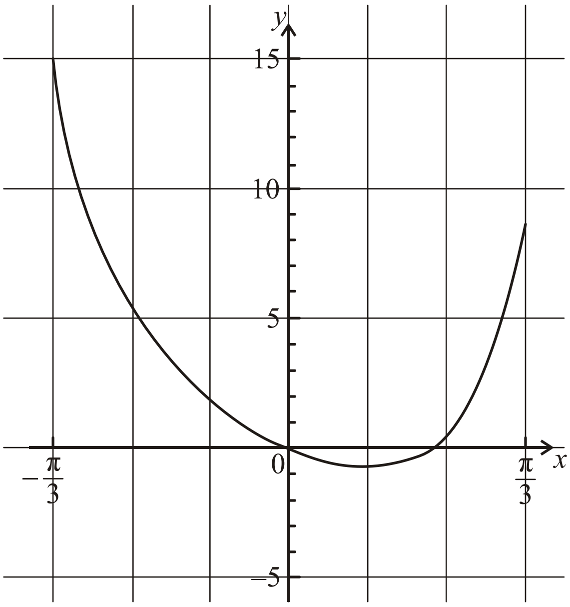
6. (a)



A1A1A1 N3

Note: *Award A1 for passing through (0, 0), A1 for correct shape, A1 for a range of approximately –1 to 15.*

(b) evidence of attempt to solve $f(x) = 1$ (M1)

e.g. line on sketch, using $\tan x = \frac{\sin x}{\cos x}$

$x = -0.207$ $x = 0.772$ A1A1 N3

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Trigonometry Test Review Paper 2 KEY

7. (a) **METHOD 1**

Using the discriminant $\Delta = 0$ (M1)

$k^2 = 4 \times 4 \times 1$

$k = 4, k = -4$ A1A1 N3

METHOD 2

Factorizing (M1)

$(2x \pm 1)^2$

$k = 4, k = -4$ A1A1 N3

(b) Evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ M1

eg $2(2\cos^2\theta - 1) + 4\cos\theta + 3$

$f(\theta) = 4\cos^2\theta + 4\cos\theta + 1$ AG N0

(c) (i) 1 A1 N1

(ii) **METHOD 1**

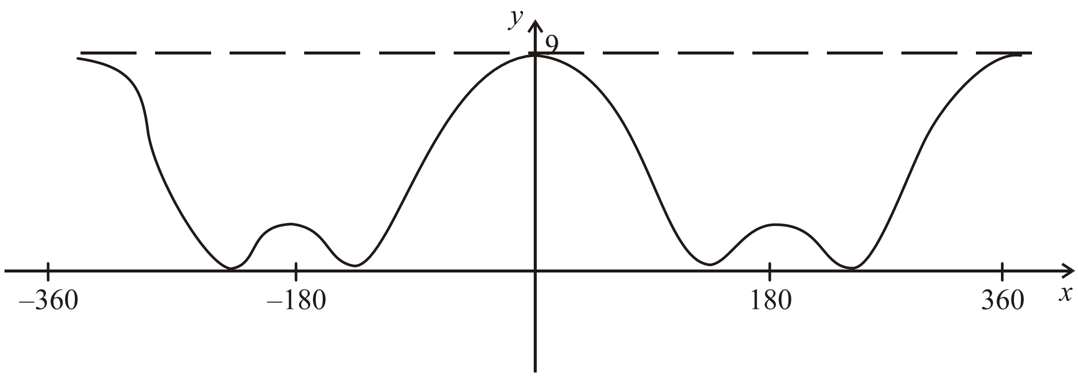
Attempting to solve for $\cos\theta$ M1

$\cos\theta = -\frac{1}{2}$ (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

METHOD 2

Sketch of $y = 4\cos^2\theta + 4\cos\theta + 1$ M1



Indicating 4 zeros (A1)

$\theta = 240, 120, -240, -120$ (correct four values only) A2 N3

(d) Using sketch (M1)

$c = 9$ A1 N2

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8. ***Note:*** *Accept* ***exact*** *answers given in terms of π.*

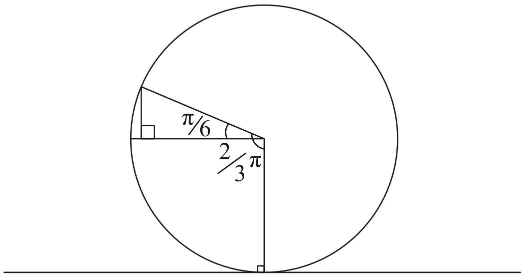
(a) Evidence of using $l = r\theta$ (M1)

arc AB = 7.85 (m) A1 N2

(b) Evidence of using $A = \frac{1}{2}r^2\theta$ (M1)

Area of sector AOB = 58.9 (m^2) A1 N2

(c) **METHOD 1**



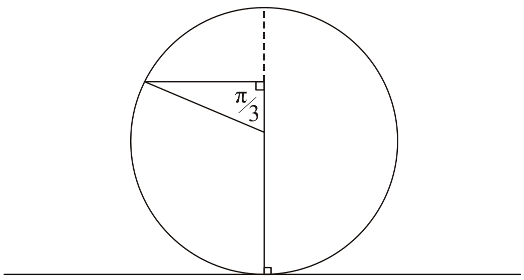
angle = $\frac{\pi}{6}$ $(30°)$ (A1)

attempt to find $15 \sin \frac{\pi}{6}$ M1

height = $15 + 15 \sin \frac{\pi}{6}$

= 22.5 (m) A1 N2

METHOD 2



angle = $\frac{\pi}{3}$ $(60°)$ (A1)

attempt to find $15 \cos \frac{\pi}{3}$ M1

height = $15 + 15 \cos \frac{\pi}{3}$

= 22.5 (m) A1 N2

(d) (i) $h\left(\frac{\pi}{4}\right)=15-15\cos\left(\frac{\pi}{2}+\frac{\pi}{4}\right)$ (M1)

$= 25.6$ (m) A1 N2

(ii) $h(0) = 15 - 15\cos\left(0+\frac{\pi}{4}\right)$ (M1)

$= 4.39$(m) A1 N2

(iii) **METHOD 1**

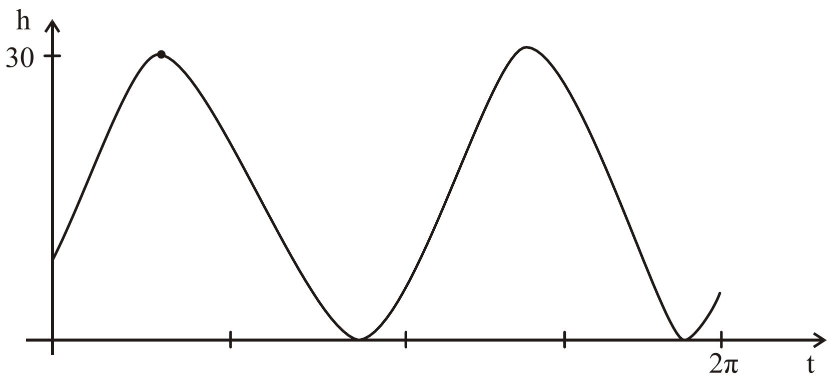
Highest point when $h = 30$ R1

$30 = 15 - 15\cos\left(2t+\frac{\pi}{4}\right)$ M1

$\cos\left(2t+\frac{\pi}{4}\right) = -1$ (A1)

$t = 1.18$ $\left(\text{accept } \frac{3\pi}{8}\right)$ A1 N2

METHOD 2



Sketch of graph of h M2
Correct maximum indicated (A1)
$t = 1.18$ A1 N2

METHOD 3

Evidence of setting $h'(t) = 0$ M1

$\sin\left(2t+\frac{\pi}{4}\right) = 0$ (A1)

Justification of maximum R1

eg reasoning from diagram, first derivative test, second derivative test

$t = 1.18$ $\left(\text{accept } \frac{3\pi}{8}\right)$ A1 N2

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9.	(a)	(i)	$10 + 4\sin 1 = 13.4$	(A1)		
		(ii)	At 2100, $t = 21$	(A1)		
			$10 + 4\sin 10.5 = 6.48$	(A1)	(N2)	3
			Note: *Award (A0)(A1) if candidates use t = 2100 leading to y = 12.6. No other **ft** allowed.*			
	(b)	(i)	14 metres	(A1)		
		(ii)	$14 = 10 + 4\sin\left(\frac{t}{2}\right) \Rightarrow \sin\left(\frac{t}{2}\right) = 1$	(M1)		
			$\Rightarrow t = \pi$ (3.14) (correct answer only)	(A1)	(N2)	3
	(c)	(i)	4	(A1)		
		(ii)	$10 + 4\sin\left(\frac{t}{2}\right) = 7$	(M1)		
			$\Rightarrow \sin\left(\frac{t}{2}\right) = -0.75$	(A1)		
			$\Rightarrow t = 7.98$	(A1)	(N3)	
		(iii)	depth < 7 from 8 –11 = 3 hours	(M1)		
			from 2030 – 2330 = 3 hours	(M1)		
			therefore, total = 6 hours	(A1)	(N3)	7

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