Trigonometry Test Review Paper 1 KEY

1. **METHOD 1**

using double-angle identity (seen anywhere) A1

e.g. $\sin 2x = 2\sin x \cos x$, $2\cos x = 2\sin x \cos x$

evidence of valid attempt to solve equation (M1)

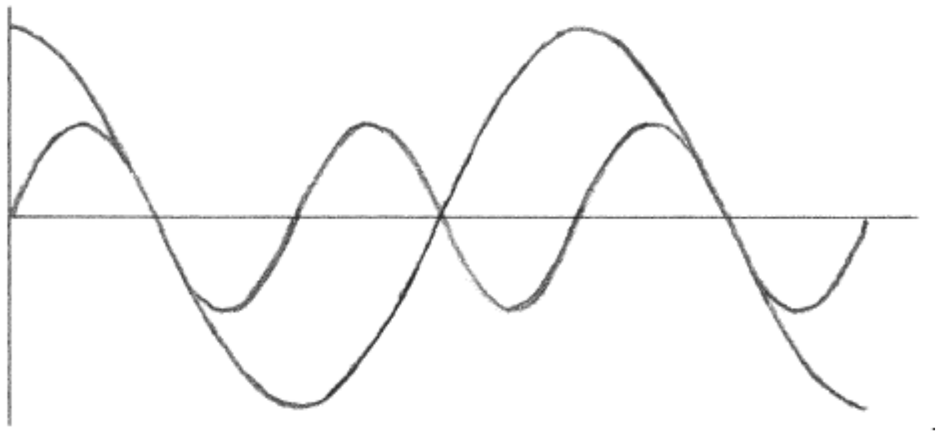
e.g. $0 = 2\sin x \cos x - 2\cos x$, $2\cos x (1 - \sin x) = 0$

$\cos x = 0$, $\sin x = 1$ A1A1

$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x = \frac{5\pi}{2}$ A1A1A1 N4

[7]

METHOD 2



A1A1M1A1

Notes: *Award A1 for sketch of sin 2x, A1 for a sketch of 2 cos x, M1 for at least one intersection point seen, and A1 for 3 approximately correct intersection points. Accept sketches drawn outside [0, 3π], even those with more than 3 intersections.*

$x = \frac{\pi}{2}, x = \frac{3\pi}{2}, x\frac{5\pi}{2}$ A1A1A1 N4

[7]

Trigonometry Test Review Paper 1 KEY

2. (a) (i) 100 (metres) A1 N1

(ii) 50 (metres) A1 N1 2

(b) (i) identifying symmetry with $h(2) = 9.5$ (M1)

subtraction A1

e.g. $100 - h(2)$, $100 - 9.5$

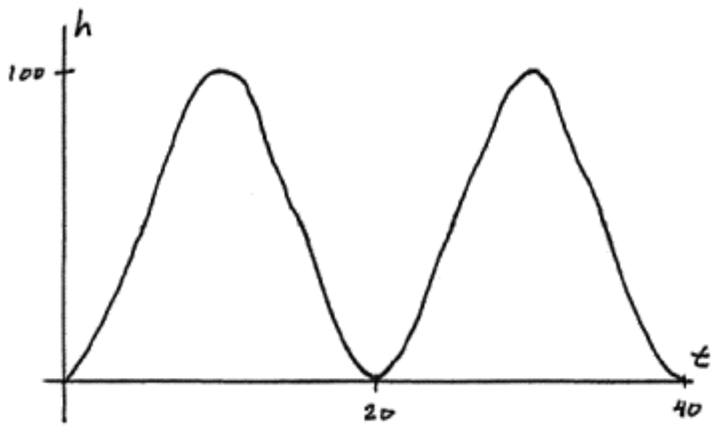
$h(8) = 90.5$ AG N0

(ii) recognizing period (M1)

e.g. $h(21) = h(1)$

$h(21) = 2.4$ A1 N2 4

(c)



A1A1A1 N3 3

Note: *Award A1 for end points (0, 0)* ***and*** *(40, 0), A1 for range $0 \le h \le 100$, A1 for approximately correct sinusoidal shape, with two cycles*

(d) evidence of a quotient involving 20, 2π or $360°$ to find b (M1)

e.g. $\frac{2\pi}{b} = 20, b = \frac{360}{20}$

$b = \frac{2\pi}{20}\left(= \frac{\pi}{10}\right)$

(accept $b = 18$ if working in degrees) A1 N2

$a = -50$, $c = 50$ A2A1 N3 5

[14]

Trigonometry Test Review Paper 1 KEY

3. (a) $\tan\theta = \frac{3}{4}\left(\text{do not accept } \frac{3}{4}x\right)$ A1 N1

(b) (i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ (A1)(A1)

correct substitution A1

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ A1 N3

(ii) correct substitution A1

e.g. $\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2, \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ A1 N1

[7]

4. $e^{2x}(\sqrt{3}\sin x + \cos x) = 0$ (A1)

$e^{2x} = 0$ not possible (seen anywhere) (A1)

simplifying

e.g. $\sqrt{3}\sin x + \cos x = 0, \sqrt{3}\sin x = -\cos x, \frac{\sin x}{-\cos x} = \frac{1}{\sqrt{3}}$ A1

EITHER

$\tan x = -\frac{1}{\sqrt{3}}$ A1

$x = \frac{5\pi}{6}$ A2 N4

OR

sketch of 30°, 60°, 90° triangle with sides 1, 2, $\sqrt{3}$ A1

work leading to $x = \frac{5\pi}{6}$ A1

verifying $\frac{5\pi}{6}$ satisfies equation A1 N4

[6]

Trigonometry Test Review Paper 1 KEY

5. (a) attempt to substitute $1 - 2\sin^2\theta$ for $\cos 2\theta$ (M1)

correct substitution A1

e.g. $4 - (1 - 2\sin^2\theta) + 5\sin\theta$

$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ AG N0

(b) evidence of appropriate approach to solve (M1)

e.g. factorizing, quadratic formula

correct working A1

e.g. $(2\sin\theta + 3)(\sin\theta + 1)$, $(2x + 3)(x + 1) = 0$, $\sin x = \dfrac{-5 \pm \sqrt{1}}{4}$

correct solution $\sin\theta = -1$ $\left(\text{do not penalise for including } \sin\theta = -\dfrac{3}{2}\right)$ (A1)

$\theta = \dfrac{3\pi}{2}$ A2 N3

[7]

6. (a) (i) $\sin 140° = p$ A1 N1

(ii) $\cos 70° = -q$ A1 N1

(b) **METHOD 1**

evidence of using $\sin^2\theta + \cos^2\theta = 1$ (M1)

e.g. diagram, $\sqrt{1 - p^2}$ (seen anywhere)

$\cos 140° = \pm\sqrt{1 - p^2}$ (A1)

$\cos 140° = -\sqrt{1 - p^2}$ A1 N2

METHOD 2

evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ (M1)

$\cos 140° = 2\cos^2 70 - 1$ (A1)

$\cos 140° = 2(-q)^2 - 1$ $(= 2q^2 - 1)$ A1 N2

(c) **METHOD 1**

$\tan 140° = \dfrac{\sin 140°}{\cos 140°} = -\dfrac{p}{\sqrt{1 - p^2}}$ A1 N1

METHOD 2

$\tan 140° = \dfrac{p}{2q^2 - 1}$ A1 N1

[6]

Trigonometry Test Review Paper 1 KEY

7.	(a)	changing $\tan x$ into $\dfrac{\sin x}{\cos x}$	A1	
		e.g. $\sin^3 x + \cos^3 x \dfrac{\sin x}{\cos x}$		
		simplifying	A1	
		e.g. $\sin x\,(\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$		
		$f(x) = \sin x$	AG	N0
	(b)	recognizing $f(2x) = \sin 2x$, seen anywhere	(A1)	
		evidence of using double angle identity $\sin(2x) = 2\sin x \cos x$, seen anywhere	(M1)	
		evidence of using Pythagoras with $\sin x = \dfrac{2}{3}$	M1	
		e.g. sketch of right triangle, $\sin^2 x + \cos^2 x = 1$		
		$\cos x = -\dfrac{\sqrt{5}}{3}\left(\text{accept } \dfrac{\sqrt{5}}{3}\right)$	(A1)	
		$f(2x) = 2\left(\dfrac{2}{3}\right)\left(-\dfrac{\sqrt{5}}{3}\right)$	A1	
		$f(2x) = -\dfrac{4\sqrt{5}}{9}$	AG	N0

[7]

8.	(a)	(i)	attempt to substitute	(M1)	
			e.g. $a = \dfrac{29-15}{2}$		
			$a = 7$ (accept $a = -7$)	A1	N2
		(ii)	period = 12	(A1)	
			$b = \dfrac{2\pi}{12}$	A1	
			$b = \dfrac{\pi}{6}$	AG	N0
		(iii)	attempt to substitute	(M1)	
			e.g. $d = \dfrac{29+15}{2}$		
			$d = 22$	A1	N2
		(iv)	$c = 3$ (accept $c = 9$ from $a = -7$)	A1	N1
			Note: *Other correct values for c can be found,* $c = 3 \pm 12k,\ k \in \mathbb{Z}$.		
	(b)		stretch takes 3 to 1.5	(A1)	
			translation maps (1.5, 29) to (4.5, 19) (so M′ is (4.5, 19))	A1	N2

(c) $g(t) = 7\cos\frac{\pi}{3}(t - 4.5) + 12$ A1A2A1 N4

Note: *Award A1 for* $\frac{\pi}{3}$, ***A2*** *for 4.5, A1 for 12.*
Other correct values for c can be found $c = 4.5 \pm 6k,\ k \in \mathbb{Z}$.

(d) translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ (A1)
horizontal stretch of a scale factor of 2 (A1)
completely correct description, in correct order A1 N3
e.g. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ then horizontal stretch of a scale factor of 2

[16]

9. (a) $p = 30$ A2 N2

(b) **METHOD 1**
Period $= \frac{2\pi}{q}$ (M2)
$= \frac{\pi}{2}$ (A1)
$\Rightarrow q = 4$ A1 N4

METHOD 2
Horizontal stretch of scale factor $= \frac{1}{q}$ (M2)
scale factor $= \frac{1}{4}$ (A1)
$\Rightarrow q = 4$ A1 N4

[6]

10. (a) Evidence of using Pythagoras (M1)
e.g. diagram, $\sin^2 x + \cos^2 x = 1$
Correct calculation (A1)
e.g. $5, \sqrt{1 - \frac{144}{169}}$
$\sin\theta = \frac{5}{13}$ A1 N

(b) Evidence of using formula for $\cos 2\theta$ (M1)
e.g. $\cos 2\theta = 2\cos^2\theta - 1$
Correct substitution/calculation A1
e.g. $2\left(-\frac{12}{13}\right)^2 - 1$
$\cos 2\theta = \frac{119}{169}$ A1 N2

(c) $\sin(\theta + \pi) = -\sin\theta = -\frac{5}{13}$ A1 N1

[7]

Trigonometry Test Review Paper 1 KEY

11. (a) Attempt to factorise (M1)

correct factors $(2\sin\theta - 1)(\sin\theta + 1) = 0$ A1

$\sin\theta = \frac{1}{2}$, $\sin\theta = -1$ A1A1 N2

(b) other solutions are 150°, 270° A1A1 N1N1

[6]

12. (a) period $= \frac{2\pi}{2} = \pi$ M1A1 N2

(b) $m = \frac{\pi}{2}$ A2 N2

[4]

13. ***Note**: Throughout this question, do **not** accept methods which involve finding θ.*

(a) Evidence of correct approach A1

eg $\sin\theta = \frac{\text{BC}}{\text{AB}}$, $\text{BC} = \sqrt{3^2 - 2^2} = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$ AG N0

(b) Evidence of using $\sin 2\theta = 2\sin\theta\cos\theta$ (M1)

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$ A1

$= \frac{4\sqrt{5}}{9}$ AG N0

(c) Evidence of using an appropriate formula for $\cos 2\theta$ M1

eg $\frac{4}{9} - \frac{5}{9}$, $2 \times \frac{4}{9} - 1$, $1 - 2 \times \frac{5}{9}$, $\sqrt{\left(1 - \frac{80}{81}\right)}$

$\cos 2\theta = -\frac{1}{9}$ A2 N2

[6]

Trigonometry Test Review Paper 1 KEY

14. (a) For using perimeter = $r + r$ + arc length (M1)

$20 = 2r + r\theta$ A1

$\theta = \dfrac{20-2r}{r}$ AG N0

(b) Finding $A = \dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)\ \left(=10r-r^2\right)$ (A1)

For setting up equation in r M1

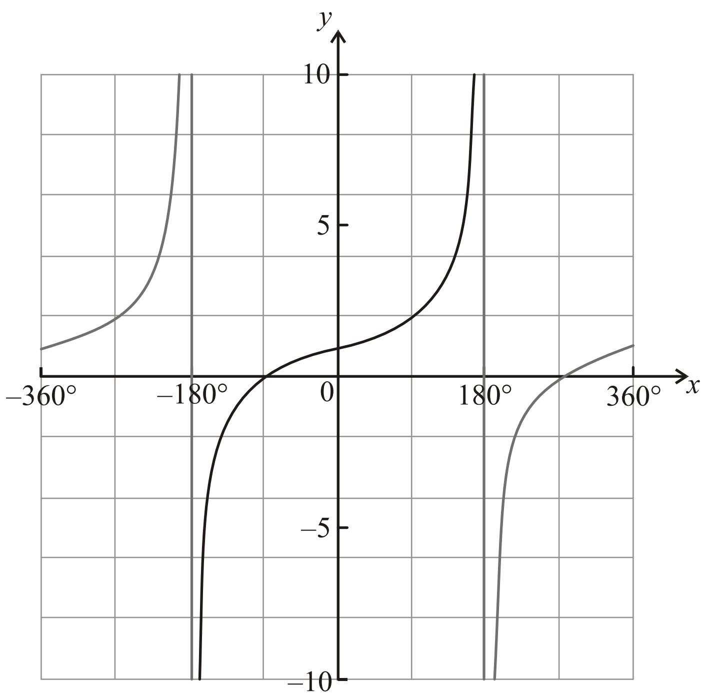
Correct simplified equation, or sketch

eg $10r - r^2 = 25$, $r^2 - 10r + 25 = 0$ (A1)

$r = 5$ cm A1 N2

[6]

15. (a)



Correct asymptotes A1A1 N2

(b) (i) Period = 360° (accept 2π) A1 N1

(ii) $f(90°) = 2$ A1 N1

(c) 270°, –90° A1A1 N1N1

***Notes**: Penalize **1 mark** for any additional values.*
*Penalize **1 mark** for correct answers given in radians* $\left(\dfrac{3\pi}{2}, -\dfrac{\pi}{2}, \text{or } 4.71, -1.57\right)$.

[6]

Trigonometry Test Review Paper 1 KEY

16. $a = 4, b = 2, c = \frac{\pi}{2} \left(\text{or } \frac{3\pi}{2} \, etc \right)$ A2A2A2 N6

[6]

17. (a) $A = \frac{1}{2} r^2 \theta$

$27 = \frac{1}{2}(1.5)r^2$ (M1)(A1)

$r^2 = 36$ (A1)

$r = 6$ cm (A1) (C4)

(b) Arc length $= r\theta = 1.5 \times 6$ (M1)

Arc length = 9 cm (A1) (C2)

Note: *Penalize a total of (1 mark) for missing units.*

[6]