Topic 5 Calculus – Paper 2 KEY

1. (a) Evidence of using $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ (M1)

eg $3e^{3t-2}$

$a(1) = 3e$ (= 8.15) A1 N2

(b) Attempt to solve $e^{3t-2} = 22.3$ (M1)

eg $(3t - 2)(\ln e) = \ln 22.3$, sketch

$t = 1.70$ A1 N2

(c) Evidence of using $s = \int v\mathrm{d}t$ (limits not required) M1

$e.g.\ \int e^{3t-2}\,\mathrm{d}t, \frac{1}{3}\left[e^{3t-2}\right]_0^1$

$\frac{1}{3}\left(e^1 - e^{-2}\right)$ $\left[= \frac{1}{3}\left(e - e^{-2}\right) = 0.861\right]$ A1 N1

[6]

2. (a) $\int_{\frac{3\pi}{2}}^{2\pi} \cos x\,\mathrm{d}x$ A1 N1

(b) Area of A = 1 A1 N1

(c) Evidence of attempting to find the area of B (M1)

eg $\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} y\,\mathrm{d}x$, – 0.134

Evidence of recognising that area B is under the curve/integral is negative (M1)

eg $-\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} y\,\mathrm{d}x, \int_{\frac{3\pi}{2}}^{\frac{4\pi}{3}} \cos x\,\mathrm{d}x, \left|\int_{\frac{4\pi}{3}}^{\frac{3\pi}{2}} \cos x\,\mathrm{d}x\right|$

Area of B = 0.134 $\left(\text{accept}\ \frac{2-\sqrt{3}}{2}\right)$ (A1)

Total Area = 1 + 0.134

= 1.13 $\left(\text{accept}\ \frac{4-\sqrt{3}}{2}\right)$ A1 N4

[6]

Topic 5 Calculus – Paper 2 KEY

3. evidence of finding intersection points (M1)

e.g. $f(x) = g(x)$, $\cos x^2 = e^x$, sketch showing intersection

$x = -1.11$, $x = 0$ (may be seen as limits in the integral) A1A1

evidence of approach involving integration and subtraction (in any order)(M1)

e.g. $\int_{-1.11}^{0} \cos x^2 - e^x$, $\int (\cos x^2 - e^x)\,dx$, $\int g - f$

area = 0.282 A2 N3

[6]

4. **METHOD 1**

evidence of antidifferentiation (M1)

e.g. $\int (10e^{2x} - 5)dx$

$y = 5e^{2x} - 5x + C$ A2A1

Note: *Award A2 for* $5e^{2x}$*, A1 for* $-5x$*. If "C" is omitted, award no further marks.*

substituting (0, 8) (M1)

e.g. $8 = 5 + C$

$C = 3$ ($y = 5e^{2x} - 5x + 3$) (A1)

substituting $x = 1$ (M1)

$y = 34.9$ ($5e^2 - 2$) A1 N4 8

METHOD 2

evidence of definite integral function expression (M2)

e.g. $\int_a^x f'(t)dt = f(x) - f(a)$, $\int_0^x (10e^{2x} - 5)$

initial condition in definite integral function expression (A2)

e.g. $\int_0^x (10e^{2t} - 5)dt = y - 8$, $\int_0^x (10e^{2x} - 5)dx + 8$

correct definite integral expression for y when $x = 1$ (A2)

e.g. $\int_0^1 (10e^{2x} - 5)dx + 8$

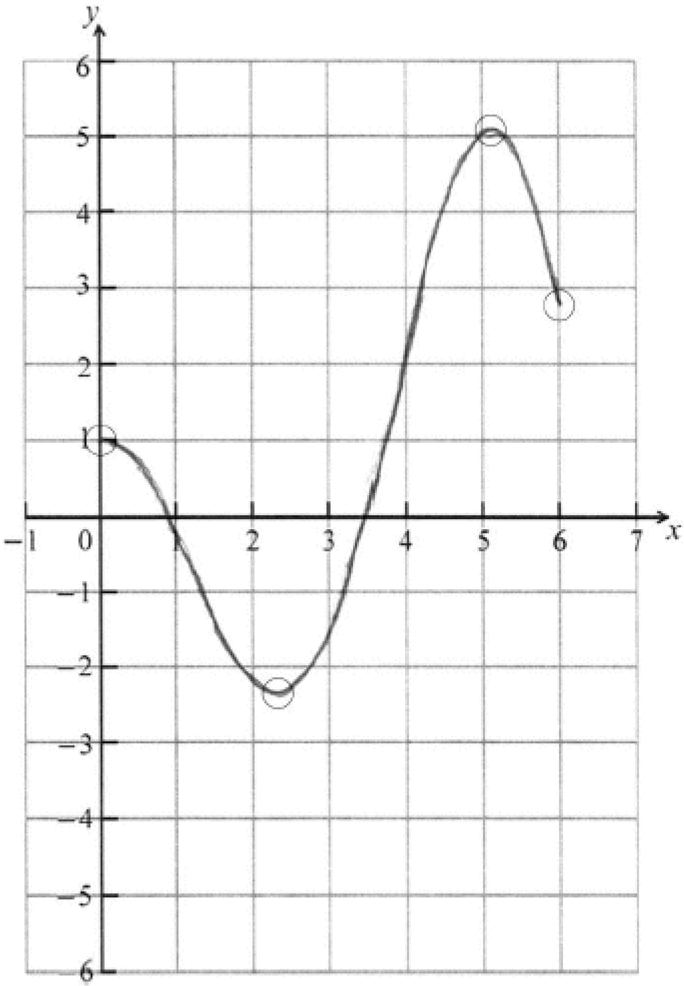
$y = 34.9$ ($5e^2 - 2$) A2 N4 8

5. (a) evidence of choosing the product rule (M1)

e.g. $x \times (-\sin x) + 1 \times \cos x$

$f'(x) = \cos x - x \sin x$ A1A1 N3

(b)



A1A1A1A1 N4

Note: *Award A1 for correct domain, $0 \le x \le 6$ with endpoints in circles,*
A1 for approximately correct shape,
A1 for local minimum in circle,
A1 for local maximum in circle.

[7]

6. evidence of integrating the acceleration function (M1)

e.g. $\int \left(\frac{1}{t} + 3\sin 2t\right) dt$

correct expression $\ln t - \frac{3}{2}\cos 2t + c$ A1A1

evidence of substituting (1, 0) (M1)

e.g. $0 = \ln 1 - \frac{3}{2}\cos 2 + c$

$c = -0.624 \left(= \frac{3}{2}\cos 2 - \ln 1 \text{ or } \frac{3}{2}\cos 2\right)$ (A1)

$v = \ln t - \frac{3}{2}\cos 2t - 0.624 \left(= \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 \text{ or } \ln t - \frac{3}{2}\cos 2t + \frac{3}{2}\cos 2 - \ln 1\right)$ (A1)

$v(5) = 2.24$ (accept the exact answer $\ln 5 - 1.5\cos 10 + 1.5\cos 2$) A1 N3

[7]

Topic 5 Calculus – Paper 2 KEY

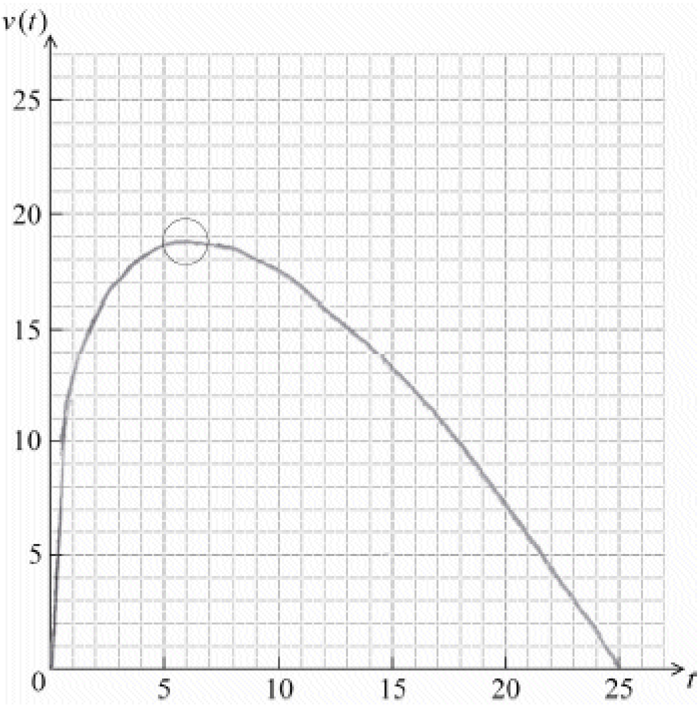
7.	(a)	2.31	A1	N1
	(b)	(i) 1.02	A1	N1
		(ii) 2.59	A1	N1
	(c)	$\int_p^q f(x)\mathrm{d}x = 9.96$	A1	N1
		split into two regions, make the area below the x-axis positive	R1R1	N2

[6]

8.	(a)	$n = 800e^0$	(A1)	
		$n = 800$	A1	N2
	(b)	evidence of using the derivative	(M1)	
		$n'(15) = 731$	A1	N2
	(c)	**METHOD 1**		
		setting up inequality (accept equation or reverse inequality) *e.g.* $n'(t) > 10\ 000$	A1	
		evidence of appropriate approach *e.g.* sketch, finding derivative	M1	
		$k = 35.1226...$	(A1)	
		least value of k is 36	A1	N2
		METHOD 2		
		$n'(35) = 9842$, **and** $n'(36) = 11208$	A2	
		least value of k is 36	A2	N2

[8]

9. (a)



A1A1A1 N3

***Note:** Award A1 for approximately correct shape, A1 for right endpoint at (25, 0) and A1 for maximum point in circle.*

(b) (i)	recognizing that d is the area under the curve	(M1)		
	e.g. $\int v(t)$			
	correct expression in terms of t, with correct limits	A2	N3	
	e.g. $d = \int_0^9 (15\sqrt{t} - 3t)\mathrm{d}t$, $d = \int_0^9 v\mathrm{d}t$			
(ii)	$d = 148.5$ (m) (accept 149 to 3 sf)	A1	N1	
				[7]
10. (a)	valid approach	R1		
	e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f			
	–0.114, 0.364 (accept (–0.114, 0.811) and (0.364, 2.13))	A1A1	N1N1	
(b)	**METHOD 1**			
	graph of g is a quadratic function	R1	N1	
	a quadratic function does not have any points of inflexion	R1	N1	
	METHOD 2			
	graph of g is concave down over entire domain	R1	N1	
	therefore no change in concavity	R1	N1	
	METHOD 3			
	$g''(x) = -144$	R1	N1	
	therefore no points of inflexion as $g''(x) \neq 0$	R1	N1	
				[5]
11. (a)	gradient is 0.6	A2	N2	
(b)	at R, $y = 0$ (seen anywhere)	A1		
	at $x = 2$, $y = \ln 5$ (= 1.609...)	(A1)		
	gradient of normal = –1.6666...	(A1)		
	evidence of finding correct equation of normal	A1		
	e.g. $y - \ln 5 = -\frac{5}{3}(x - 2)$, $y = -1.67x + c$			
	$x = 2.97$ (accept 2.96)	A1		
	coordinates of R are (2.97, 0)	N3		
				[7]
12. (a)	$f'(x) = -\sin 2x \times 2$ $(= -2\sin 2x)$	A1A1	N2	
	Note: *Award A1 for 2, A1 for sin 2x.*			
(b)	$g'(x) = 3 \times \frac{1}{3x-5}\left(= \frac{3}{3x-5}\right)$	A1A1	N2	
	Note: *Award A1 for 3, A1 for* $\frac{1}{3x-5}$.			
(c)	evidence of using product rule	(M1)		
	$h'(x) = (\cos 2x)\left(\frac{3}{3x-5}\right) + \ln(3x - 5)(-2\sin 2x)$	A1	N2	
				[6]

Topic 5 Calculus – Paper 2 KEY

13. substituting $x = 1$, $y = 3$ into $f(x)$ (M1)

$3 = p + q$ A1

finding derivative (M1)

$f'(x) = 2px + q$ A1

correct substitution, $2p + q = 8$ A1

$p = 5$, $q = -2$ A1A1 N2N2

[7]

14. **METHOD 1**

correct expression for **second** side, using area = 525 (A1)

e.g. let AB = x, AD = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side (M1)

e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $\frac{525}{x}\times 3+\frac{525}{x}\times 3+11x+3x, \frac{525}{\text{AB}}\times 3+\frac{525}{\text{AB}}\times 3+11\text{AB}+3\text{AB}, \frac{3150}{x}+14x$

EITHER

sketch of cost function (M1)

identifying minimum point (A1)

e.g. marking point on graph, $x = 15$

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = \frac{-1575}{x^2}+\frac{-1575}{x^2}+14$

setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $\frac{-3150}{x^2}+14=0$

minimum cost is 420 (dollars) A1 N4

METHOD 2

correct expression for **second** side, using area = 525 (A1)

e.g. let AD = x, AB = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side (M1)

e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $3\left(x+x+\frac{525}{x}\right)+\frac{525}{x}\times 11, 3\left(\text{AD}+\text{AD}+\frac{525}{\text{AD}}\right)+\frac{525}{\text{AD}}\times 11, 6x+\frac{7350}{x}$

EITHER

sketch of cost function (M1)

identifying minimum point (A1)

e.g. marking point on graph, $x = 35$

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = 6 - \frac{7350}{x^2}$

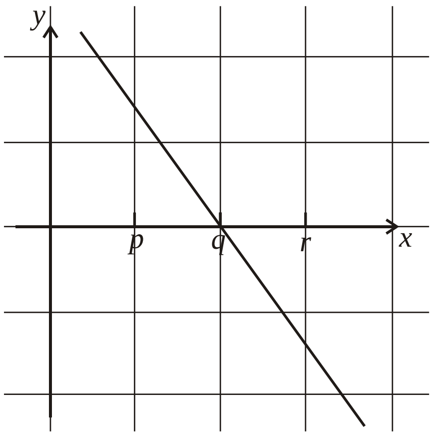
setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $6 - \frac{7350}{x^2} = 0$

minimum cost is 420 (dollars) A1 N4

[7]

15. (a)



A1A1 N2

Note: *Award A1 for ne.g.ative gradient throughout, A1 for x-intercept of q. It need not be linear.*

(b)

		x-coordinate
(i)	Maximum point on *f*	*r*
(ii)	Inflexion point on *f*	*q*

(i) A1 N1
(ii) A1 N1

(c) **METHOD 1**

Second derivative is zero, second derivative changes sign. R1R1 N2

METHOD 2

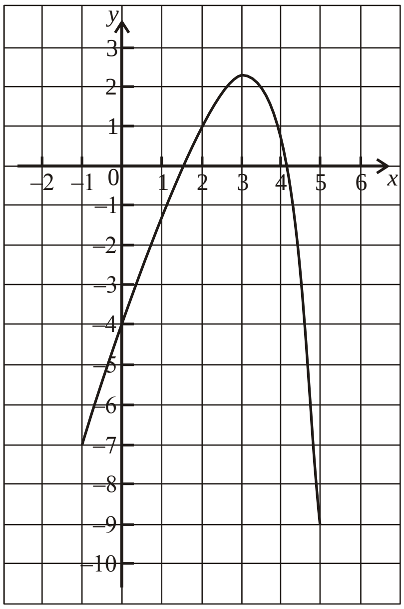
There is a maximum on the graph of the first derivative. R2 N2

[6]

16. (a) intercepts when $f(x) = 0$ (M1)

(1.54, 0) (4.13, 0) (accept $x = 1.54$ $\quad x = 4.13$) A1A1 N3

(b)



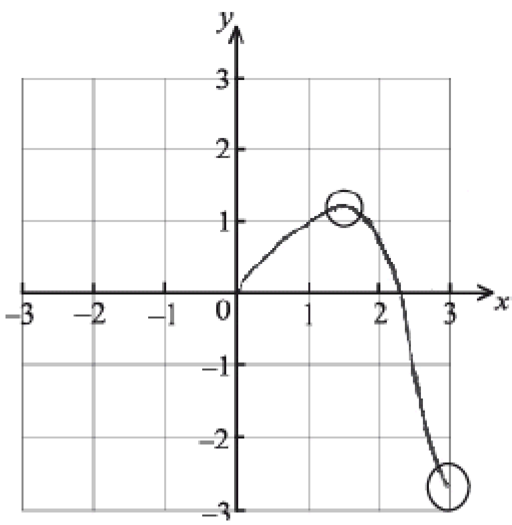
A1A1A1 N3

Note: *Award A1 for passing through approximately (0, – 4), A1 for correct shape, A1 for a range of approximately –9 to 2.3.*

(c) gradient is 2 A1 N1

[7]

17. (a)



A1A2 N3

Notes: *Award **A1** for correct domain, $0 \le x \le 3$.*
*Award **A2** for approximately correct shape, with local maximum in circle 1 and right endpoint in circle 2.*

(b) $a = 2.31$ A1 N1

(c) evidence of using $V = \pi\int[f(x)]^2\,\mathrm{d}x$ (M1)

fully correct integral expression A2

e.g. $V = \pi\int_0^{2.31}[x\cos(x - \sin x)]^2\mathrm{d}x, V = \pi\int_0^{2.31}[f(x)]^2\mathrm{d}x$

$V = 5.90$ A1 N2

[8]

18. (a) Using the chain rule (M1)

$f'(x) = (2\cos(5x - 3))5$ $(= 10\cos(5x - 3))$ A1

$f''(x) = -(10\sin(5x - 3))5$

$= -50\sin(5x - 3)$ A1A1 N2

Note: *Award A1 for sin (5x – 3), A1 for –50.*

(b) $\int f(x)\mathrm{d}x = -\frac{2}{5}\cos(5x - 3) + c$ A1A1 N2

Note: *Award A1 for cos(5x – 3), A1 for* $-\frac{2}{5}$.

[6]

19. **Note:** In this question, do not penalize absence of units.

(a) (i) $s = \int(40 - at)\mathrm{d}t$ (M1)

$s = 40t - \frac{1}{2}at^2 + c$ (A1)(A1)

substituting $s = 100$ when $t = 0$ $(c = 100)$ (M1)

$s = 40t - \frac{1}{2}at^2 + 100$ A1 N5

(ii) $s = 40t - \frac{1}{2}at^2$ A1 N1

(b) (i) stops at station, so $v = 0$ (M1)

$t = \frac{40}{a}$ (seconds) A1 N2

(ii) evidence of choosing formula for s from (a) (ii) (M1)

substituting $t = \frac{40}{a}$ (M1)

e.g. $40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}$

setting up equation M1

e.g. $500 = s$, $500 = 40 \times \frac{40}{a} - \frac{1}{2}a \times \frac{40^2}{a^2}, 500 = \frac{1600}{a} - \frac{800}{a}$

evidence of simplification to an expression which obviously leads to $a = \frac{8}{5}$ A1

e.g. $500a = 800$, $5 = \frac{8}{a}$, $1000a = 3200 - 1600$

$a = \frac{8}{5}$ AG N0

(c) **METHOD 1**

$v = 40 - 4t$, stops when $v = 0$
$40 - 4t = 0$ (A1)
$t = 10$ A1

substituting into expression for s M1

$s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$

$s = 200$ A1
since $200 < 500$ (allow ***FT*** on their s, if $s < 500$) R1
train stops before the station AG N0

METHOD 2

from (b) $t = \frac{40}{4} = 10$ A2
substituting into expression for s
e.g. $s = 40 \times 10 - \frac{1}{2} \times 4 \times 10^2$ M1
$s = 200$ A1
since $200 < 500$, R1
train stops before the station AG N0

METHOD 3

a is deceleration A2
$4 > \frac{8}{5}$ A1
so stops in shorter time (A1)
so less distance travelled R1
so stops before station AG N0

[17]

20. (a) B, D A1A1 N2 2

(b) (i) $f'(x) = -2x\mathrm{e}^{-x^2}$ A1A1 N2

Note: *Award A1 for* e^{-x^2} *and A1 for* $-2x$.

(ii) finding the derivative of $-2x$, *i.e.* -2 (A1)
evidence of choosing the product rule (M1)

e.g. $-2\mathrm{e}^{-x^2} - 2x \times -2x\mathrm{e}^{-x^2}$

$-2\mathrm{e}^{-x^2} + 4x^2\mathrm{e}^{-x^2}$ A1

$f''(x) = (4x^2 - 2)\,\mathrm{e}^{-x^2}$ AG N0 5

(c) valid reasoning R1
e.g. $f''(x) = 0$
attempting to solve the equation (M1)
e.g. $(4x^2 - 2) = 0$, sketch of $f''(x)$

$p = 0.707\left(= \frac{1}{\sqrt{2}}\right)$, $q = -0.707\ \left(= -\frac{1}{\sqrt{2}}\right)$ A1A1 N3 4

(d) evidence of using second derivative to test values on either side of POI M1

e.g. finding values, reference to graph of f'', sign table

correct working A1A1

e.g. finding any two correct values either side of POI,

checking sign of f'' on either side of POI

reference to sign change of $f''(x)$ R1 N0 4

[15]

21. (a) substituting (0, 13) into function M1

e.g. $13 = Ae^0 + 3$

$13 = A + 3$ A1

$A = 10$ AG N0

(b) substituting into $f(15) = 3.49$ A1

e.g. $3.49 = 10e^{15k} + 3$, $0.049 = e^{15k}$

evidence of solving equation (M1)

e.g. sketch, using ln

$k = -0.201$ $\left(\text{accept} \frac{\ln 0.049}{15}\right)$ A1 N2

(c) (i) $f(x) = 10e^{-0.201x} + 3$

$f'(x) = 10e^{-0.201x} \times -0.201$ $(= -2.01e^{-0.201x})$ A1A1A1 N3

Note: *Award A1 for* $10e^{-0.201x}$, *A1 for* $\times -0.201$, *A1 for the derivative of 3 is zero.*

(ii) valid reason with reference to derivative R1 N1

e.g. $f'(x) < 0$, derivative always negative

(iii) $y = 3$ A1 N1

(d) finding limits 3.8953…, 8.6940… (seen anywhere) A1A1

evidence of integrating and subtracting functions (M1)

correct expression A1

e.g. $\int_{3.90}^{8.69} g(x) - f(x)\mathrm{d}x, \int_{3.90}^{8.69} [(-x^2 + 12x - 24) - (10e^{-0.201x} + 3)]\mathrm{d}x$

area = 19.5 A2 N4

[16]

22. (a) (i) –1.15, 1.15 A1A1 N2

(ii) recognizing that it occurs at P and Q (M1)

e.g. $x = -1.15$, $x = 1.15$

$k = -1.13$, $k = 1.13$ A1A1 N3

(b) evidence of choosing the product rule (M1)
e.g. $uv' + vu'$

derivative of x^3 is $3x^2$ (A1)

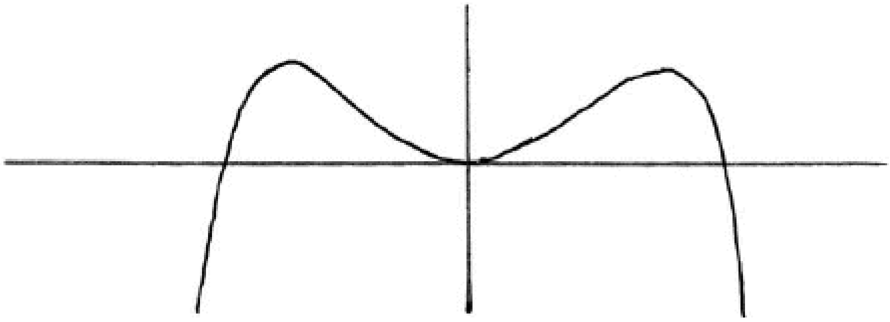
derivative of $\ln(4 - x^2)$ is $\dfrac{-2x}{4-x^2}$ (A1)

correct substitution A1

e.g. $x^3 \times \dfrac{-2x}{4-x^2} + \ln(4-x^2) \times 3x^2$

$g'(x) = \dfrac{-2x^4}{4-x^2} + 3x^2 \ln(4-x^2)$ AG N0

(c)



A1A1 N2

(d) $w = 2.69$, $w < 0$ A1A2 N2

[14]

23. (a) evidence of valid approach (M1)
e.g. $f(x) = 0$, graph

$a = -1.73$, $b = 1.73$ $(a = -\sqrt{3}, b = \sqrt{3})$ A1A1 N3

(b) attempt to find max (M1)
e.g. setting $f'(x) = 0$, graph

$c = 1.15$ (accept (1.15, 1.13)) A1 N2

(c) attempt to substitute either limits or the function into formula M1

e.g. $V = \pi\int_0^c [f(x)]^2 \mathrm{d}x, \pi\int [x\ln(4-x^2)]^2, \pi\int_0^{1.149...} y^2 \mathrm{d}x$

$V = 2.16$ A2 N2

(d) valid approach recognizing 2 regions (M1)
e.g. finding 2 areas

correct working (A1)

e.g. $\int_0^{-1.73...} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x; -\int_{-1.73...}^{0} f(x)\mathrm{d}x + \int_0^{1.149...} f(x)\mathrm{d}x$

area = 2.07 (accept 2.06) A2 N3

[12]

Topic 5 Calculus – Paper 2 KEY

24. (a) attempt to expand (M1)

$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$ A1 N2

(b) evidence of substituting x + *h* (M1)

correct substitution A1

e.g. $f'(x) = \lim_{h\to 0}\dfrac{(x+h)^3 - 4(x+h) + 1 - (x^3 - 4x + 1)}{h}$

simplifying A1

e.g. $\dfrac{(x^3 + 3x^2h + 3xh^2 + h^2 - 4x - 4h + 1 - x^3 + 4x - 1)}{h}$

factoring out *h* A1

e.g. $\dfrac{h(3x^2 + 3xh + h^2 - 4)}{h}$

$f'(x) = 3x^2 - 4$ AG N0

(c) $f'(1) = -1$ (A1)

setting up an appropriate equation M1

e.g. $3x^2 - 4 = -1$

at Q, $x = -1$, $y = 4$ (Q is (–1, 4)) A1A1 N3

(d) recognizing that *f* is decreasing when $f'(x) < 0$ R1

correct values for *p* and *q* (but do not accept $p = 1.15$, $q = -1.15$) A1A1 N1N1

e.g. $p = -1.15$, $q = 1.15$; $\pm\dfrac{2}{\sqrt{3}}$; an interval such as $-1.15 \le x \le 1.15$

(e) $f'(x) \ge -4$, $y \ge -4$, $[-4, \infty[$ A2 N2

[15]

25. (a) finding the limits $x = 0$, $x = 5$ (A1)

integral expression A1

e.g. $\int_0^5 f(x)\,\mathrm{d}x$

area = 52.1 A1 N2

(b) evidence of using formula $v = \int \pi y^2\,\mathrm{d}x$ (M1)

correct expression A1

e.g. volume $= \pi\int_0^5 x^2(x-5)^4\,\mathrm{d}x$

volume = 2340 A2 N2

(c) area is $\int_0^a x(a - x)\,\mathrm{d}x$ A1

$= \left[\dfrac{ax^2}{2} - \dfrac{x^3}{3}\right]_0^a$ A1A1

substituting limits (M1)

e.g. $\dfrac{a^3}{2} - \dfrac{a^3}{3}$

setting expression equal to area of *R* (M1)

correct equation A1

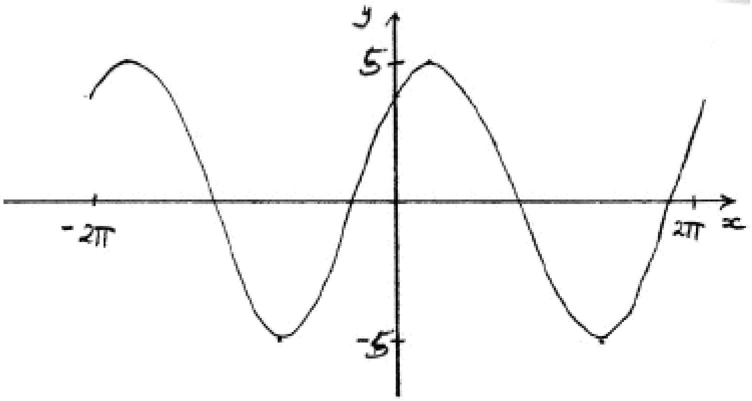
e.g. $\dfrac{a^2}{2} - \dfrac{a^3}{3} = 52.1$, $a^3 = 6 \times 52.1$,

$a = 6.79$ A1 N3

[14]

Topic 5 Calculus – Paper 2 KEY

26. (a)



A1A1A1 N3

> ***Note:*** *Award A1 for approximately sinusoidal shape, A1 for end points approximately correct, (–2π, 4), (2π, 4) A1 for approximately correct position of graph, (y-intercept (0, 4) maximum to right of y-axis).*

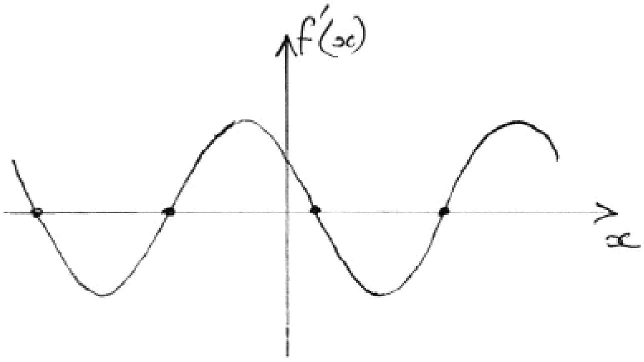
(b) (i) 5 — A1 N1

(ii) 2π (6.28) — A1 N1

(iii) –0.927 — A1 N1

(c) $f(x) = 5 \sin (x + 0.927)$ (accept $p = 5$, $q = 1$, $r = 0.927$) — A1A1A1 N3

(d) evidence of correct approach — (M1)
e.g. max/min, sketch of $f'(x)$ indicating roots

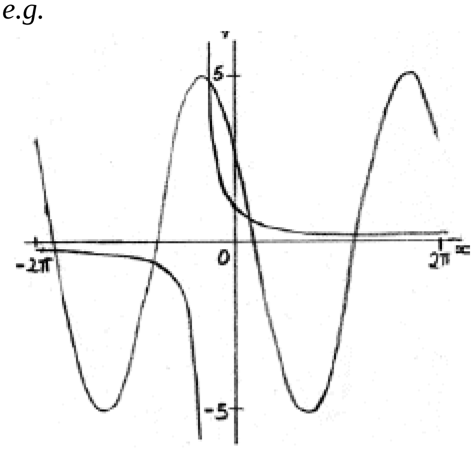


one 3 s.f. value which rounds to one of –5.6, –2.5, 0.64, 3.8 — A1 N2

(e) $k = –5$, $k = 5$ — A1A1 N2

(f) **METHOD 1**

graphical approach (but must involve derivative functions) — M1
e.g.



each curve — A1A1
$x = 0.511$ — A2 N2

METHOD 2

$g'(x) = \dfrac{1}{x+1}$ A1

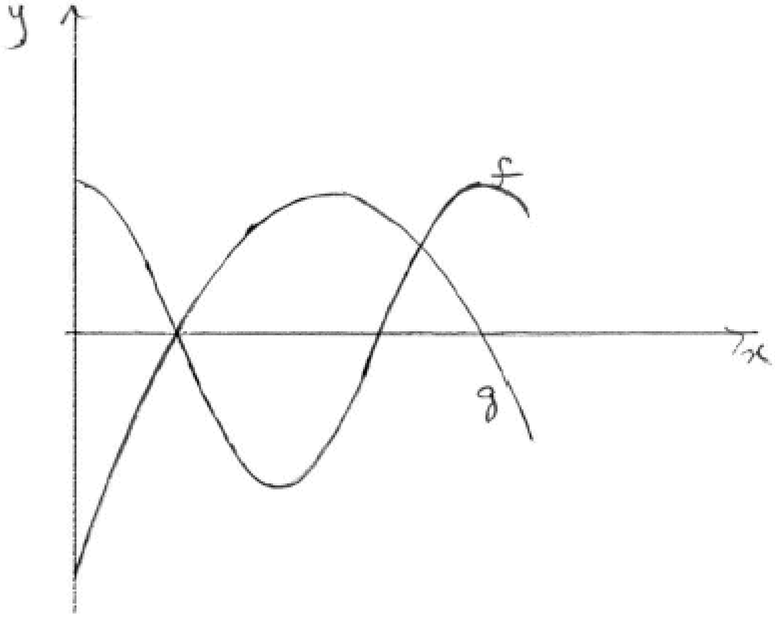
$f'(x) = 3\cos x - 4\sin x$ $(5\cos(x + 0.927))$ A1

evidence of attempt to solve $g'(x) = f'(x)$ M1

$x = 0.511$ A2 N2

[18]

27. (a)



A1A1A1 N3

Note: *Award A1 for f being of sinusoidal shape, with 2 maxima and one minimum,*
A1 for g being a parabola opening down,
A1 for **two** *intersection points in approximately correct position.*

(b) (i) (2,0) (accept $x = 2$) A1 N1

(ii) period = 8 A2 N2

(iii) amplitude = 5 A1 N1

(c) (i) (2, 0), (8, 0) (accept $x = 2$, $x = 8$) A1A1 N1N1

(ii) $x = 5$ (must be an equation) A1 N1

(d) **METHOD 1**

intersect when $x = 2$ and $x = 6.79$ (may be seen as limits of integration) A1A1

evidence of approach (M1)

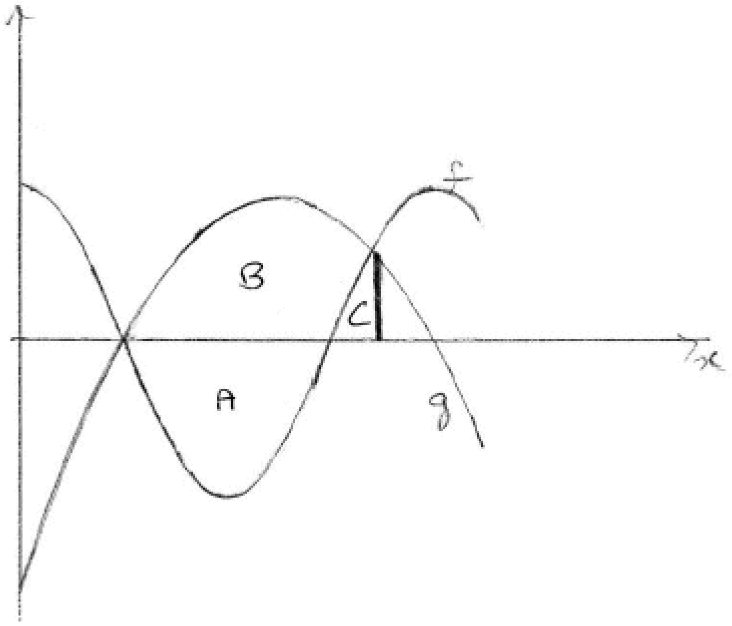
e.g. $\int g - f, \int f(x)\mathrm{d}x - \int g(x)\mathrm{d}x, \int_2^{6.79}\left((-0.5x^2 + 5x - 8 - \left(5\cos\frac{\pi}{4}x\right)\right)$

area = 27.6 A2 N3

METHOD 2

intersect when $x = 2$ and $x = 6.79$ (seen anywhere) A1A1

evidence of approach using a sketch of g and f, or $g - f$. (M1)



e.g. area $A + B - C$, $12.7324 + 16.0938 - 1.18129...$
area = 27.6 A2 N3

[15]

28. (a) (i) intersection points $x = 3.77$, $x = 8.30$ (may be seen as the limits) (A1)(A1)
approach involving subtraction and integrals (M1)

fully correct expression A2

e.g. $\int_{3.77}^{8.30} ((-4\cos(0.5x)+2)-(\ln(3x-2)+1))\mathrm{d}x$,

$\int_{3.77}^{8.30} g(x)\mathrm{d}x - \int_{3.77}^{8.30} f(x)\mathrm{d}x$ N5

(ii) $A = 6.46$ A1 N1

(b) (i) $f'(x) = \frac{3}{3x-2}$ A1A1 N2

Note: *Award A1 for numerator (3), A1 for denominator (3x − 2), but penalize 1 mark for additional terms.*

(ii) $g'(x) = 2\sin(0.5x)$ A1A1 N2

Note: *Award A1 for 2, A1 for sin (0.5x), but penalize 1 mark for additional terms.*

(c) evidence of using derivatives for gradients (M1)
correct approach (A1)
e.g. $f'(x) = g'(x)$, points of intersection
$x = 1.43$, $x = 6.10$ A1A1 N2N2

[14]

Topic 5 Calculus – Paper 2 KEY

29. (a) evidence of using the product rule M1

$f'(x) = e^x(1 - x^2) + e^x(-2x)$ A1A1

Note: *Award A1 for $e^x(1-x^2)$, A1 for $e^x(-2x)$.*

$f'(x) = e^x(1 - 2x - x^2)$ AG N0

(b) $y = 0$ A1 N1

(c) at the local maximum or minimum point

$f'(x) = 0 \quad (e^x(1 - 2x - x^2) = 0)$ (M1)

$\Rightarrow 1 - 2x - x^2 = 0$ (M1)

$r = -2.41$ $s = 0.414$ A1A1 N2N2

(d) $f'(0) = 1$ A1

gradient of the normal = –1 A1

evidence of substituting into an equation for a straight line (M1)

correct substitution A1

e.g. $y - 1 = -1(x - 0)$, $y - 1 = -x$, $y = -x + 1$

$x + y = 1$ AG N0

(e) (i) intersection points at $x = 0$ and $x = 1$ (may be seen as the limits) (A1)

approach involving subtraction and integrals (M1)

fully correct expression A2 N4

e.g. $\int_0^1 \left(e^x\left(1-x^2\right)-(1-x)\right)dx, \int_0^1 f(x)dx - \int_0^1 (1-x)dx$

(ii) area $R = 0.5$ A1 N1

[17]

30. (a) correctly finding the derivative of e^{2x}, *i.e.* $2e^{2x}$ A1

correctly finding the derivative of $\cos x$, *i.e.* $-\sin x$ A1

evidence of using the product rule, seen anywhere M1

e.g. $f'(x) = 2e^{2x}\cos x - e^{2x}\sin x$

$f'(x) = e^{2x}(2\cos x - \sin x)$ AG N0

(b) evidence of finding $f(0) = 1$, seen anywhere A1

attempt to find the gradient of f (M1)

e.g. substituting $x = 0$ into $f'(x)$

value of the gradient of f A1

e.g. $f'(0) = 2$, equation of tangent is $y = 2x + 1$

gradient of normal = $-\frac{1}{2}$ (A1)

$y - 1 = -\frac{1}{2}x \quad \left(y = -\frac{1}{2}x + 1\right)$ A1 N3

(c) (i) evidence of equating correct functions M1

e.g. $e^{2x}\cos x = -\frac{1}{2}x + 1$, sketch showing intersection of graphs

$x = 1.56$ A1 N1

(ii) evidence of approach involving subtraction of integrals/areas (M1)

e.g. $\int [f(x) - g(x)]dx, \int f(x)dx$ – area under trapezium

fully correct integral expression A2

e.g. $\int_0^{1.56} \left[e^{2x} \cos x - \left(-\frac{1}{2}x + 1 \right) \right] dx, \int_0^{1.56} e^{2x} \cos x dx - 0.951...$

area = 3.28 A1 N2

[14]

31. (a) Curve intersects y-axis when $x = 0$ (A1)

Gradient of tangent at y-intercept = 2 A1

$\Rightarrow$ gradient of $N = -\frac{1}{2}$ (= –0.5) A1

Finding y-intercept, 2.5 A1

Therefore, equation of N is $y = -0.5x + 2.5$ AG N0

(b) N intersects curve when $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ A1

Solving equation (M1)

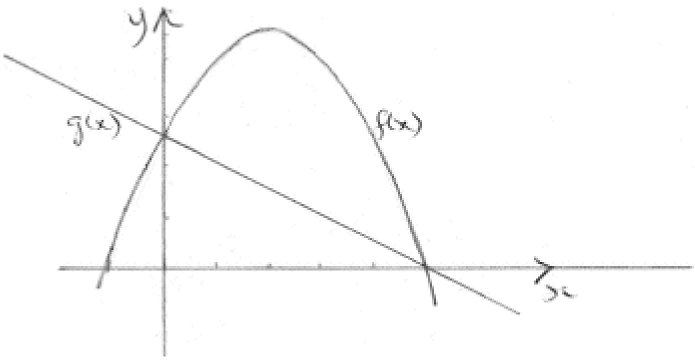
e.g. sketch, factorising

$\Rightarrow x = 0$ or $x = 5$ A1

Other point when $x = 5$ (R1)

$x = 5 \Rightarrow y = 0$ (so other point (5, 0)) A1 N2

(c)



Using appropriate method, with subtraction/correct expression, **correct** limitsM1A1

e.g. $\int_0^5 f(x)dx - \int_0^5 g(x)dx, \int_0^5 (-0.5x^2 + 2.5x)dx$

Area = 10.4 A2 N2

[13]

32. (a) (i) $p = 1, q = 5$ (or $p = 5, q = 1$) A1A1 N2

(ii) $x = 3$ (must be an equation) A1 N1

(b) $y = (x - 1)(x - 5)$

$= x^2 - 6x + 5$ (A1)

$= (x - 3)^2 - 4$ (accept $h = 3, k = -4$) A1A1 N3

(c) $\frac{dy}{dx} = 2(x - 3) \ (= 2x - 6)$ A1A1 N2

(d) When $x = 0$, $\frac{dy}{dx} = -6$ (A1)

$y - 5 = -6(x - 0)$ ($y = -6x + 5$ or equivalent) A1 N2

[10]

Topic 5 Calculus – Paper 2 KEY

33. (a) π (3.14) (accept $(\pi, 0)$, $(3.14, 0)$) A1 N1

(b) (i) For using the product rule (M1)

$f'(x) = e^x \cos x + e^x \sin x = e^x(\cos x + \sin x)$ A1A1 N3

(ii) At B, $f'(x) = 0$ A1 N1

(c) $f''(x) = e^x \cos x - e^x \sin x + e^x \sin x + e^x \cos x$ A1A1

$= 2e^x \cos x$ AG N0

(d) (i) At A, $f''(x) = 0$ A1 N1

(ii) Evidence of setting up **their** equation (may be seen in part (d)(i)) A1

eg $2e^x \cos x = 0$, $\cos x = 0$

$x = \frac{\pi}{2}\ (=1.57)$, $y = e^{\frac{\pi}{2}}\ (=4.81)$ A1A1

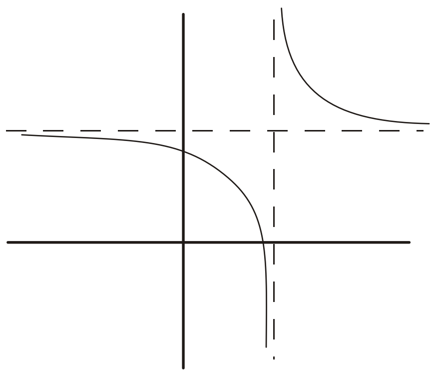
Coordinates are $\left(\frac{\pi}{2}, e^{\frac{\pi}{2}}\right)$ $(1.57, 4.81)$ N2

(e) (i) $\int_0^{\pi} e^x \sin x \,dx$ or $\int_0^{\pi} f(x)\,dx$ A2 N2

(ii) Area = 12.1 A2 N2

[15]

34. (a)



A1A1A1 N3

***Notes**: Award A1 for **both** asymptotes shown.*
The asymptotes need not be labelled.
*Award A1 for the left branch in **approximately** correct position,*
*A1 for the right branch in **approximately** correct position.*

(b) (i) $y = 3$, $x = \frac{5}{2}$ (must be equations) A1A1 N2

(ii) $x = \frac{14}{6}$ $\left(\frac{7}{3} \text{ or } 2.33, \text{ also accept} \left(\frac{14}{6}, 0\right)\right)$ A1 N1

(iii) $y = \frac{14}{6}$ $(y = 2.8)$ $\left(\text{accept} \left(0, \frac{14}{5}\right) \text{ or } (0, 2.8)\right)$ A1 N1

(c) (i) $\int \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx = 9x +$

$3\ln(2x-5) - \frac{1}{2(2x-5)} + C$ A1A1A1

A1A1 N5

(ii) Evidence of using $V = \int_a^b \pi y^2 dx$ (M1)

Correct expression A1

eg $\int_3^a \pi \left(3 + \frac{1}{2x-5}\right)^2 dx, \pi \int_3^a \left(9 + \frac{6}{2x-5} + \frac{1}{(2x-5)^2}\right) dx,$

$\left[9x + 3\ln(2x-5) - \frac{1}{2(2x-5)}\right]_3^a$

Substituting $\left(9a + 3\ln(2a-5) - \frac{1}{2(2a-5)}\right) - \left(27 + 3\ln 1 - \frac{1}{2}\right)$ A1

Setting up an equation (M1)

$9a - \frac{1}{2(2a-5)} - 27 + \frac{1}{2} + 3\ln(2a-5) - 3\ln 1 = \left(\frac{28}{3} + 3\ln 3\right)$

Solving gives $a = 4$ A1 N2

[17]

35. (a) (i) $p = 2$ A1 N1

(ii) $q = 1$ A1 N1

(b) (i) $f(x) = 0$ (M1)

$2 - \frac{3x}{x^2 - 1} = 0$ $(2x^2 - 3x - 2 = 0)$ A1

$x = -\frac{1}{2}$ $x = 2$

$\left(-\frac{1}{2}, 0\right)$ A1 N2

(ii) Using $V = \int_a^b \pi y^2 dx$ (limits not required) (M1)

$V = \int_{-\frac{1}{2}}^{0} \pi \left(2 - \frac{3x}{x^2 - 1}\right)^2 dx$ A2

$V = 2.52$ A1 N2

(c) (i) Evidence of appropriate method — M1

eg Product or quotient rule

Correct derivatives of $3x$ and $x^2 - 1$ — A1A1

Correct substitution — A1

$$eg \quad \frac{-3(x^2-1)-(-3x)(2x)}{(x^2-1)^2}$$

$$f'(x) = \frac{-3x^2+3+6x^2}{(x^2-1)^2}$$ A1

$$f'(x) = \frac{3x^2+3}{(x^2-1)^2} = \frac{3(x^2+1)}{(x^2-1)^2}$$ AG N0

(ii) **METHOD 1**

Evidence of using $f'(x) = 0$ at max/min — (M1)

$3(x^2 + 1) = 0$ $(3x^2 + 3 = 0)$ — A1

no (real) solution — R1

Therefore, no maximum or minimum. — AG N0

METHOD 2

Evidence of using $f'(x) = 0$ at max/min — (M1)

Sketch of $f'(x)$ with good asymptotic behaviour — A1

Never crosses the x-axis — R1

Therefore, no maximum or minimum. — AG N0

METHOD 3

Evidence of using $f'(x) = 0$ at max/min — (M1)

Evidence of considering the sign of $f'(x)$ — A1

$f'(x)$ is an increasing function ($f'(x) > 0$, always) — R1

Therefore, no maximum or minimum. — AG N0

(d) For using integral — (M1)

Area = $\int_0^a g(x)\,dx$ $\left(\text{or} \int_0^a f'(x)\,dx \text{ or} \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx\right)$ — A1

Recognizing that $\int_0^a g(x)\,dx = f(x)\Big|_0^a$ — A2

Setting up equation (seen anywhere) — (M1)

Correct equation — A1

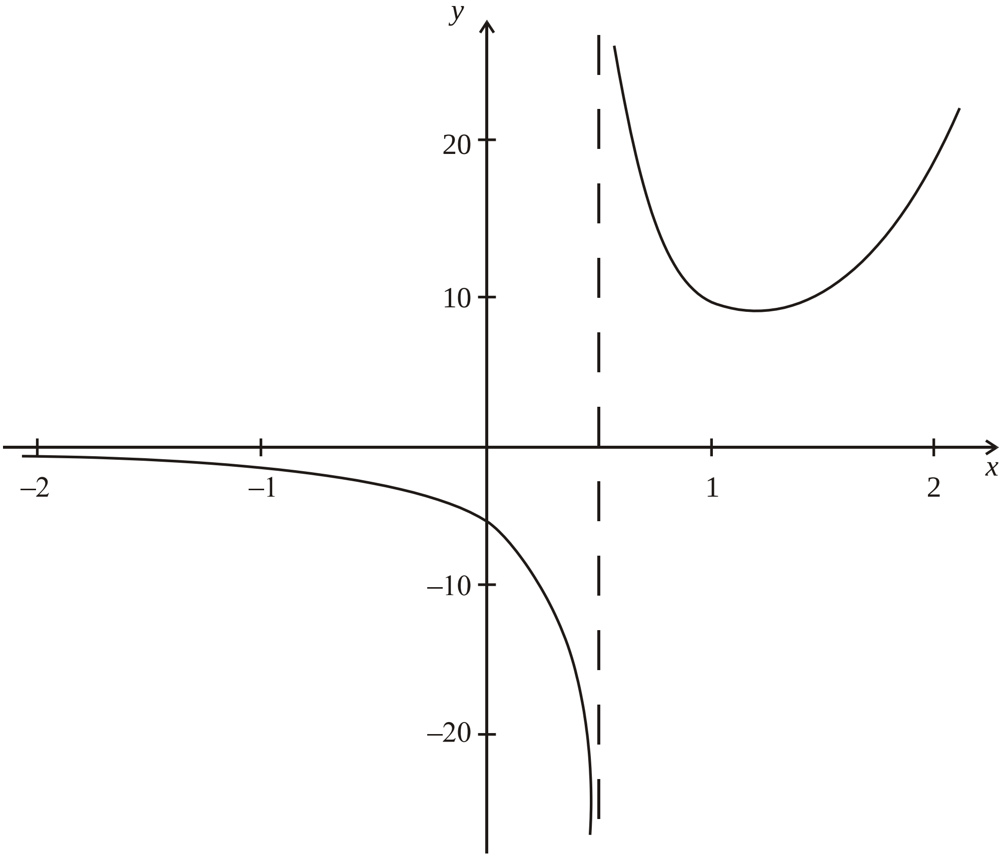
$$eg \quad \int_0^a \frac{3x^2+3}{(x^2-1)^2}\,dx = 2, \left[2 - \frac{3a}{a^2-1}\right] - [2-0] = 2,\ 2a^2 + 3a - 2 = 0$$

$a = \frac{1}{2}$ $\quad a = -2$

$a = \frac{1}{2}$ — A1 N2

[24]

36. (a)



A1A1A1 N3

Note: *Award A1 for the left branch asymptotic to the x-axis and crossing the y-axis,*
A1 for the right branch approximately the correct shape,
A1 for a vertical asymptote at approximately $x = \frac{1}{2}$.

(b) (i) $x = \frac{1}{2}$ (must be an equation) A1 N1

(ii) $\int_0^2 f(x)\,\mathrm{d}x$ A1 N1

(iii) Valid reason R1 N1

eg reference to area undefined or discontinuity

Note: *GDC reason* ***not*** *acceptable.*

(c) (i) $V = \pi\int_1^{1.5} f(x)^2\,\mathrm{d}x$ A2 N2

(ii) $V = 105$ (accept $33.3\,\pi$) A2 N2

(d) $f'(x) = 2e^{2x-1} - 10(2x-1)^{-2}$ A1A1A1A1 N4

(e) (i) $x = 1.11$ (accept $(1.11, 7.49)$) A1 N1

(ii) $p = 0$, $q = 7.49$ (accept $0 \le k < 7.49$) A1A1 N2

[17]

Topic 5 Calculus – Paper 2 KEY

37. (a) (i) $f'(x) = -\frac{3}{2}x + 1$ A1A1 N2

(ii) For using the derivative to find the gradient of the tangent (M1)

$f'(2) = -2$ (A1)

Using negative reciprocal to find the gradient of the normal $\left(\frac{1}{2}\right)$ M1

$y - 3 = \frac{1}{2}(x - 2)$ $\left(\text{or } y = \frac{1}{2}x + 2\right)$ A1 N3

(iii) Equating $-\frac{3}{4}x^2 + x + 4 = \frac{1}{2}x + 2$ (or sketch of graph) M1

$3x^2 - 2x - 8 = 0$ (A1)

$(3x + 4)(x - 2) = 0$

$x = -\frac{4}{3}$ $(= -1.33)$ (accept $\left(-\frac{4}{3}, \frac{4}{3}\right)$ or $x = -\frac{4}{3}, x = 2$) A1 N2

(b) (i) Any **completely** correct expression (accept absence of dx) A2

eg $\int_{-1}^{2}\left(-\frac{3}{4}x^2 + x + 4\right)\mathrm{d}x, \left[-\frac{1}{4}x^3 + \frac{1}{2}x^2 + 4x\right]_{-1}^{2}$ N2

(ii) Area = $\frac{45}{4}$ $(= 11.25)$ (accept 11.3) A1 N1

(iii) Attempting to **use** the formula for the volume (M1)

eg $\int_{-1}^{2}\pi\left(-\frac{3}{4}x^2 + x + 4\right)\mathrm{d}x, \pi\int_{-1}^{2}\left(-\frac{3}{4}x^2 + x + 4\right)^2\mathrm{d}x$ A2 N3

(c) $\int_{1}^{k} f(x)\,\mathrm{d}x = \left[-\frac{1}{4}x^3 + \frac{1}{2}x^2 + 4x\right]_{1}^{k}$ A1A1A1

Note: *Award A1 for* $-\frac{1}{4}x^3$*, A1 for* $\frac{1}{2}x^2$*, A1 for 4x.*

Substituting $\left(-\frac{1}{4}k^3 + \frac{1}{2}k^2 + 4k\right) - \left(-\frac{1}{4} + \frac{1}{2} + 4\right)$ (M1)(A1)

$= -\frac{1}{4}k^3 + \frac{1}{2}k^2 + 4k - 4.25$ A1 N3

[21]

38. (a) **METHOD 1**

Attempting to interchange x and y (M1)

Correct expression $x = 3y – 5$ (A1)

$f^{-1}(x) = \frac{x+5}{3}$ A1 N3

METHOD 2

Attempting to solve for x in terms of y (M1)

Correct expression $x = \frac{y+5}{3}$ (A1)

$f^{-1}(x) = \frac{x+5}{3}$ A1 N3

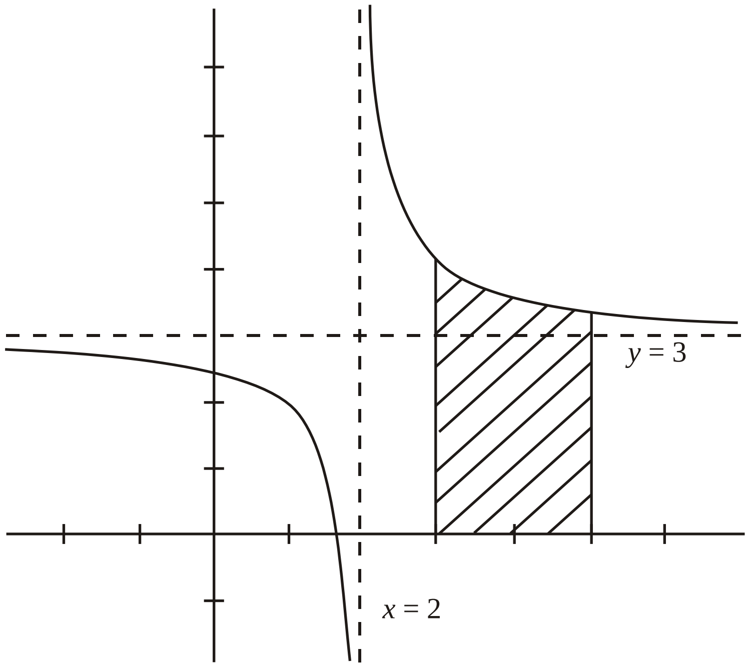
(b) For correct composition $(g^{-1} \circ f)(x) = (3x – 5) + 2$ (A1)

$(g^{-1} \circ f)(x) = 3x – 3$ A1 N2

(c) $\frac{x+3}{3} = 3x - 3 \ (x + 3 = 9x - 9)$ (A1)

$x = \frac{12}{8}$ A1 N2

(d) (i)



A1A1A1 N3

Note: *Award A1 for approximately correct x and y intervals, A1 for two branches of correct shape, A1 for both asymptotes.*

(ii) (Vertical asymptote) $x = 2$, (Horizontal asymptote) $y = 3$ A1A1 N2

(Must be equations)

(e) (i) $3x + \ln(x - 2) + C$ $\quad (3x + \ln|x - 2| + C)$ A1A1 N2

(ii) $\left[3x + \ln(x-2)\right]_3^5$ (M1)

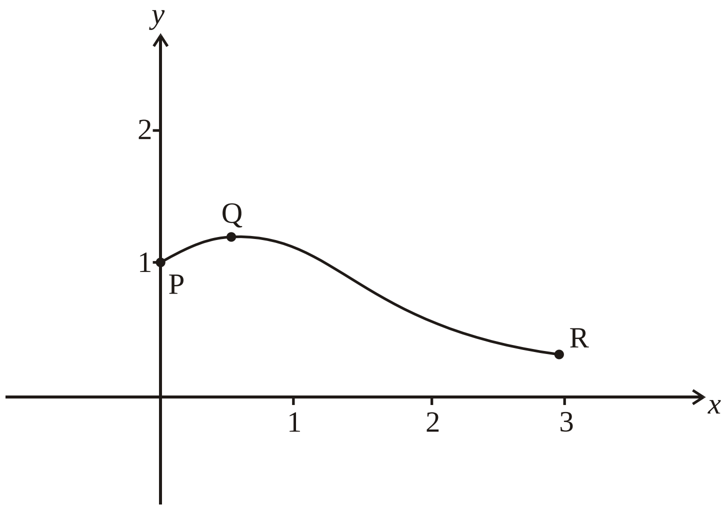
$= (15 + \ln 3) - (9 + \ln 1)$ A1

$= 6 + \ln 3$ A1 N2

(f) Correct shading (see graph). A1 N1

[18]

39. (a)



A1A1A1 N3

***Note**: Award A1 for the shape of the curve,*
A1 for correct domain,
*A1 for labelling **both** points P and Q in approximately correct positions.*

(b) (i) Correctly finding derivative of $2x + 1$ *ie* 2 (A1)

Correctly finding derivative of e^{-x} *ie* $-e^{-x}$ (A1)

Evidence of using the product rule (M1)

$f'(x) = 2e^{-x} + (2x + 1)(-e^{-x})$ A1

$= (1 - 2x)e^{-x}$ AG N0

(ii) At **Q**, $f'(x) = 0$ (M1)

$x = 0.5$, $y = 2e^{-0.5}$ A1A1

Q is $(0.5, 2e^{-0.5})$ N3

(c) $1 \le k < 2e^{-0.5}$ A2 N2

(d) Using $f''(x) = 0$ at the point of inflexion M1

$e^{-x}(-3 + 2x) = 0$

This equation has only one root. R1

So f has only one point of inflexion. AG N0

(e) At R, $y = 7e^{-3}$ (= 0.34850 ...) (A1)

Gradient of (PR) is $\frac{7e^{-3}-1}{3}\,(=-0.2172)$ (A1)

Equation of (PR) is $g(x) = \left(\frac{7e^{-3}-1}{3}\right)x+1\,(=-0.2172x+1)$ A1

Evidence of appropriate method, involving subtraction of integrals or areas M2
Correct limits/endpoints A1

eg $\int_0^3 (f(x)-g(x))\,dx$, area under curve – area under PR

Shaded area is $\int_0^3\left((2x+1)e^{-x}-\left(\frac{7e^{-3}-1}{3}x+1\right)\right)dx$

= 0.529 A1 N4

[21]

40. (a) (i) $f'(x) = -x + 2$ A1

(ii) $f'(0) = 2$ A1 2

(b) Gradient of tangent at y-intercept = $f'(0) = 2$

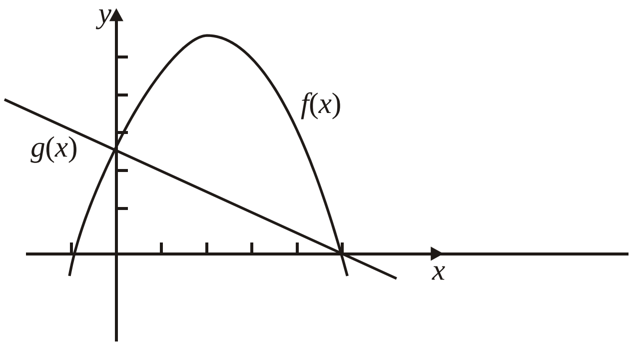
$\Rightarrow$ gradient of normal = $\frac{1}{2}$ (= –0.5) A1

Finding y-intercept is 2.5 A1
Therefore, equation of the normal is

$y - 2.5 = \sim(x - 0)$ $(y - 2.5 = -0.5x)$ M1
$(y = -0.5x + 2.5$ (AG) 3

(c) (i) **EITHER**
solving $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ (M1)A1
$\Rightarrow x = 0$ or $x = 5$ A1 2

OR



M1

Curves intersect at $x = 0$, $x = 5$ (A1)
So solutions to $f(x) = g(x)$ are $x = 0$, $x = 5$ A1 2

OR

$\Rightarrow 0.5x^2 - 2.5x = 0$ (A1)
$\Rightarrow -0.5x(x - 5) = 0$ M1
$\Rightarrow x = 0$ or $x = 5$ A1 2

(ii) Curve and normal intersect when $x = 0$ or $x = 5$ (M2)
Other point is when $x = 5$
$\Rightarrow y = -0.5(5) + 2.5 = 0$ (so other point (5, 0) A1 2

(d) (i) Area = $\int_0^5 (f(x) - g(x))\mathrm{d}x \left(\text{or} \int_0^5 (-0.5x^2 + 2x + 2.5)\mathrm{d}x - \frac{1}{2} \times 5 \times 2.5 \right)$

A1A1A1 3

***Note**: Award (A1) for the integral, (A1) for both correct limits on the integral, and (A1) for the difference.*

(ii) Area = Area under curve – area under line ($A = A_1 - A_2$) (M1)

(A1) = $\frac{50}{3}, A_2 = \frac{25}{4}$

Area = $\frac{50}{3} - \frac{25}{4} = \frac{125}{12}$ (or 10.4 (3sf) A1 2

[16]

41. (a) (i) $p = (10x + 2) - (1 + e^{2x})$ A2 2

***Note**: Award (A1) for $(l + e^{2x}) - (10x + 2)$.*

(ii) $\frac{\mathrm{d}p}{\mathrm{d}x} = 10 - 2e^{2x}$ A1A1

$\frac{\mathrm{d}p}{\mathrm{d}x} = 0$ $(10 - 2e^{2x} = 0)$ M1

$x = \frac{1\text{n}\,5}{2}$ (= 0.805) A1 4

(b) (i) **METHOD 1**

$x = 1 + e^{2x}$ M1

$1\text{n}(x - 1) = 2y$ A1

$f^{-1}(x) = \frac{1\text{n}(x-1)}{2} \left(\text{Allow } y = \frac{1\text{n}(x-1)}{2} \right)$ A1 3

METHOD 2

$y - 1 = e^{2x}$ A1

$\frac{\ln(y-1)}{2} = x$ M1

$f^{-1}(x) = \frac{1\text{n}(x-1)}{2} \left(\text{Allow } y = \frac{1\text{n}(x-1)}{2} \right)$ A1 3

(ii) $a = \frac{1\text{n}(5-1)}{2} \left(= \frac{1}{2} 1\text{n}2^2 \right)$ M1

$= \frac{1}{2} \times 21\text{n}2$ A1

$= 1\text{n}\,2$ AG 2

(c) Using $V = \int_a^b \pi y^2 \,\mathrm{d}x$ (M1)

Volume = $\int_0^{\ln 2} \pi(1 + e^{2x})^2 \mathrm{d}x \quad \left(\text{or} \int_0^{0.805} \pi(1 + e^{2x})^2 \mathrm{d}x \right)$ A2 3

[14]

Topic 5 Calculus – Paper 2 KEY

42. (a) $x = 1$ (A1) 1

(b) Using quotient rule (M1)

Substituting correctly $g'(x) = \dfrac{(x-1)^2(1)-(x-2)[2(x-1)]}{(x-1)^4}$ A1

$= \dfrac{(x-1)-(2x-4)}{(x-1)^3}$ (A1)

$= \dfrac{3-x}{(x-1)^3}$ (Accept $a = 3$, $n = 3$) A1 4

(c) Recognizing at point of inflexion $g''(x) = 0$ M1

$x = 4$ A1

Finding corresponding y-value $= \dfrac{2}{9} = 0.222$ *ie* $\mathrm{P}\left(4, \dfrac{2}{9}\right)$ A1 3

[8]

43. (a) (i) $p = -2 \quad q = 4$ (or $p = 4, q = -2$) (A1)(A1) (N1)(N1)

(ii) $y = a(x+2)(x-4)$

$8 = a(6+2)(6-4)$ (M1)

$8 = 16a$

$a = \dfrac{1}{2}$ (A1) (N1)

(iii) $y = \dfrac{1}{2}(x+2)(x-4)$

$y = \dfrac{1}{2}(x^2-2x-8)$

$y = \dfrac{1}{2}x^2 - x - 4$ (A1) (N1) 5

(b) (i) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x-1$ (A1) (N1)

(ii) $x-1=7$ (M1)

$x = 8, y = 20$ (P is (8, 20)) (A1)(A1) (N2) 4

(c) (i) when $x = 4$, gradient of tangent is $4 – 1 = 3$ (may be implied) (A1)

gradient of normal is $-\frac{1}{3}$ (A1)

$y-0=-\frac{1}{3}(x-4)$ $\left(y=-\frac{1}{3}x+\frac{4}{3}\right)$ (A1) (N3)

(ii) $\frac{1}{2}x^2-x-4=-\frac{1}{3}x+\frac{4}{3}$ (or sketch/graph) (M1)

$\frac{1}{2}x^2-\frac{2}{3}x-\frac{16}{3}=0$

$3x^2-4x-32=0$ (may be implied) (A1)

$(3x+8)(x-4)=0$

$x=-\frac{8}{3}$ or $x=4$

$x=-\frac{8}{3}$ (–2.67) (A1) (N2) 6

[15]

44. (a) $x = 1$ (A1)

EITHER

The gradient of $g(x)$ goes from positive to negative (R1)

OR

$g(x)$ goes from increasing to decreasing (R1)

OR

when $x=1$, $g''(x)$ is negative (R1) 2

(b) $-3<x<-2$ **and** $1<x<3$ (A1)

$g'(x)$ is negative (R1) 2

(c) $x=-\frac{1}{2}$ (A1)

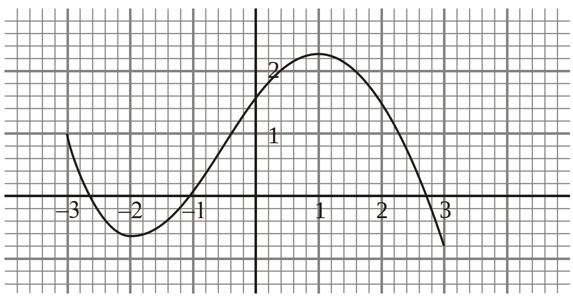
EITHER

$g''(x)$ changes from positive to negative (R1)

OR

concavity changes (R1) 2

(d)



(A3) 3

[9]

Topic 5 Calculus – Paper 2 KEY
45.

Note: *There are many approaches possible. However, there must be some evidence of their method.*

Area = $\int_0^k \sin 2x\,dx$ (must be seen somewhere) (A1)

Using area = 0.85 (must be seen somewhere) (M1)

EITHER

Integrating $\left[\frac{-1}{2}\cos 2x\right]_0^k$

$\left(=\frac{-1}{2}\cos 2\text{k}+\frac{1}{2}\cos 0\right)$ (A1)

Simplifying $\frac{-1}{2}\cos 2k + 0.5$ (A1)

Equation $\frac{-1}{2}\cos 2k + 0.5 = 0.85$ $(\cos 2k = -0.7)$

OR

Evidence of using trial and error on a GDC (M1)(A1)

Eg $\int_0^{\frac{\pi}{2}} \sin 2x\,dx = 0.5$, $\frac{\pi}{2}$ too small etc

OR

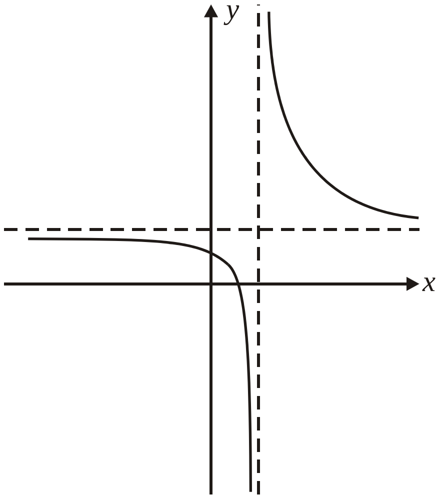
Using GDC and solver, starting with $\int_0^k \sin 2x\,dx - 0.85 = 0$(M1)(A1)

THEN

$k = 1.17$ (A2) (N3)

[6]

46. (a)



(A1)(A1) 2

Note: *Award (A1) for a second branch in approximately the correct position, and (A1) for the second branch having positive x and y intercepts. Asymptotes need not be drawn.*

(b) (i) x-intercept = $\frac{1}{2}\left(\text{Accept}\left(\frac{1}{2},0\right), x=\frac{1}{2}\right)$ (A1)

y-intercept = 1 (Accept (0, 1), y = 1) (A1)

(ii) horizontal asymptote $y = 2$ (A1)
vertical asymptote $x = 1$ (A1) 4

(c) (i) $f'(x) = 0 - (x-1)^{-2}$ $\left(= \frac{-1}{(x-1)^2}\right)$ (A2)

(ii) no maximum / minimum points.
since $\frac{-1}{(x-1)^2} \neq 0$ (R1) 3

(d) (i) $2x + \ln(x-1) + c$ (accept $\ln|x-1|$) (A1)(A1)(A1)

(ii) $A = \int_2^4 f(x)\,dx \left(\text{Accept} \int_2^4 \left(2 + \frac{1}{x-1}\right)dx, [2x + \ln(x-1)]_2^4\right)$ (M1)(A1)

Notes: *Award (A1) for **both** correct limits.*
Award (M0)(A0) for an incorrect function.

(iii) $A = [2x + \ln(x-1)]_2^4$
$= (8 + \ln 3) - (4 + \ln 1)$ (M1)
$= 4 + \ln 3 (= 5.10$, to 3 sf) (A1)
(N2) 7

[16]

47. (a) (i) $a = 1 - \pi$ (accept $(1-\pi, 0)$) (A1)

(ii) $b = 1 + \pi$ (accept $(1+\pi, 0)$) (A1) 2

(b) (i) $\int_{-2.14}^{1} h(x)\,dx - \int_1^2 h(x)\,dx$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \left|\int_1^2 h(x)\,dx\right|$ (M1)(A1)(A1)

OR

$\int_{-2.14}^{1} h(x)\,dx + \int_2^1 h(x)\,dx$ (M1)(A1)(A1)

(ii) $5.141... - (-0.1585...)$
$= 5.30$ (A2) 5

(c) (i) $y = 0.973$ (A1)

(ii) $-0.240 < k < 0.973$ (A3) 4

[11]

48. (a) $y = 0$ (A1) 1

(b) $f'(x) = \frac{-2x}{(1+x^2)^2}$ (A1)(A1)(A1) 3

(c) $\frac{6x^2 - 2}{(1+x^2)^3} = 0$ (or sketch of $f'(x)$ showing the maximum) (M1)

$6x^2 - 2 = 0$ (A1)

$x = \pm\sqrt{\frac{1}{3}}$ (A1)

$x = \frac{-1}{\sqrt{3}} (= -0.577)$ (A1) (N4) 4

(d) $\int_{-0.5}^{0.5} \frac{1}{1+x^2}\,dx \left(= 2\int_{0}^{0.5} \frac{1}{1+x^2}\,dx = 2\int_{-0.5}^{0} \frac{1}{1+x^2}\,dx \right)$ (A1)(A1) 2

[10]

49. (a) $x = 4$ (A1)

g'' changes sign at $x = 4$ or concavity changes (R1) 2

(b) $x = 2$ (A1)

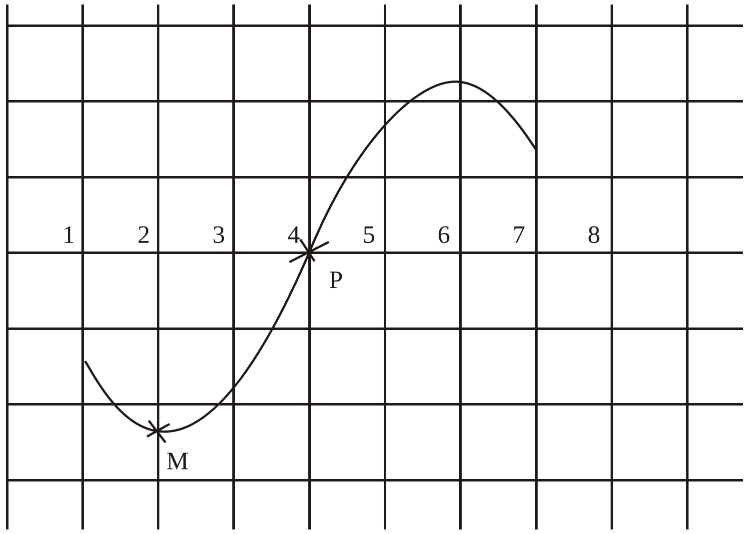
EITHER

g' goes from negative to positive (R1)

OR

$g'(2) = 0$ **and** $g''(2)$ is positive (R1) 2

(c)



(A2)(A1)(A1) 4

Note: *Award (A2) for a suitable cubic curve through (4, 0), (A1) for M at x = 2, (A1) for P at (4, 0).*

[8]

50. (a) (i) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ (A1)

therefore $\cos\left(-\frac{\pi}{4}\right) + \sin\left(-\frac{\pi}{4}\right) = 0$ (AG)

(ii) $\cos x + \sin x = 0 \Rightarrow 1 + \tan x = 0$

$\Rightarrow \tan x = -1$ (M1)

$x = \frac{3\pi}{4}$ (A1)

Note: *Award (A0) for 2.36.*

OR

$x = \frac{3\pi}{4}$ (G2) 3

(b) $y = e^x(\cos x + \sin x)$

$\frac{dy}{dx} = e^x(\cos x + \sin x) + e^x(-\sin x + \cos x)$ (M1)(A1)(A1) 3

$= 2e^x \cos x$

(c) $\frac{dy}{dx} = 0$ for a turning point $\Rightarrow 2e^x \cos x = 0$ (M1)

$\Rightarrow \cos x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{2} \Rightarrow a = \frac{\pi}{2}$ (A1)

$y = e^{\frac{\pi}{2}}(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}) = e^{\frac{\pi}{2}}$

$b = e^{\frac{\pi}{2}}$ (A1) 4

Note: *Award (M1)(A1)(A0)(A0) for a = 1.57, b = 4.81.*

(d) At D, $\frac{d^2y}{dx^2} = 0$ (M1)

$2e^x \cos x - 2e^x \sin x = 0$ (A1)

$2e^x (\cos x - \sin x) = 0$

$\Rightarrow \cos x - \sin x = 0$ (A1)

$\Rightarrow x = \frac{\pi}{4}$ (A1)

$\Rightarrow y = e^{\frac{\pi}{4}}(\cos \frac{\pi}{4} + \sin \frac{\pi}{4})$ (A1)

$= \sqrt{2}\, e^{\frac{\pi}{4}}$ (AG) 5

(e) Required area = $\int_0^{\frac{3}{4}} e^x (\cos x + \sin x)dx$ (M1)

= 7.46 sq units (G1)

OR

Area = 7.46 sq units (G2) 2

Note*: Award (M1)(G0) for the answer 9.81 obtained if the calculator is in degree mode.*

[17]