# Summation Notation

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$$

Examples:

$$\sum_{i=0}^{5} \left(\frac{2}{i+1}\right)^i = \left(\frac{2}{1}\right)^0 + \left(\frac{2}{2}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{4}\right)^3 + \left(\frac{2}{5}\right)^4 + \left(\frac{2}{6}\right)^5 = 1 + 1 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{2}{5}\right)^4 + \left(\frac{1}{3}\right)^5 \approx 2.599$$

$$\sum_{i=1}^{6} \left(\frac{2}{i}\right)^{i-1} = \left(\frac{2}{1}\right)^0 + \left(\frac{2}{2}\right)^1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{4}\right)^3 + \left(\frac{2}{5}\right)^4 + \left(\frac{2}{6}\right)^5 = 1 + 1 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{2}{5}\right)^4 + \left(\frac{1}{3}\right)^5 \approx 2.599$$

**In Exercises 1–8, write the given series in expanded form.**

**1.** $\displaystyle\sum_{k=2}^{6} k$

**2.** $\displaystyle\sum_{k=1}^{10} 5k$

**3.** $\displaystyle\sum_{k=1}^{5} \frac{1}{k}$

**4.** $\displaystyle\sum_{n=3}^{7} (4n - 7)$

**5.** $\displaystyle\sum_{n=0}^{\infty} 3^{1-n}$

**6.** $\displaystyle\sum_{j=2}^{\infty} j(-1)^j$

**7.** $\displaystyle\sum_{t=-2}^{2} 4^t$

**8.** $\displaystyle\sum_{s=0}^{6} |13 - 3s|$

**In Exercises 9–16, express the given series using sigma notation.**

**9.** $4 + 8 + 12 + 16 + 20$

**10.** $1 + 2 + 4 + 8 + 16 + 32$

**11.** $5 + 9 + 13 + \cdots + 101$

**12.** $2 + 4 + 6 + 8 + \cdots + 200$

**13.** $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$

**14.** $\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \cdots$

**15.** $\sin x + \sin 2x + \sin 3x + \cdots$

**16.** $48 + 24 + 12 + 6 + \cdots$

**17.** Show that $\displaystyle\sum_{t=1}^{4} \log t = \log 24$.

**18.** Show that $\displaystyle\sum_{k=1}^{4} k \log 2 = \log 2^{10}$.

**19.** Evaluate: $\displaystyle\sum_{k=1}^{100} \cos k\pi$

**20.** Evaluate: $\displaystyle\sum_{k=1}^{50} \sin\left(k \cdot \frac{\pi}{2}\right)$

Answers:

1. $2 + 3 + 4 + 5 + 6$

3. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

5. $3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

7. $\frac{1}{16} + \frac{1}{4} + 1 + 4 + 16$

9. $\displaystyle\sum_{k=1}^{5} 4k$

11. $\displaystyle\sum_{k=1}^{25} 4k + 1$

13. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$

15. $\displaystyle\sum_{k=1}^{\infty} \sin kx$

17. $\displaystyle\sum_{t=1}^{4} \log t = \log 1 + \log 2 + \log 3 + \log 4 = \log(1 \cdot 2 \cdot 3 \cdot 4) = \log 24$

19. $0$