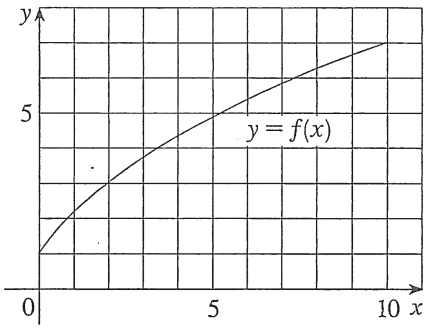
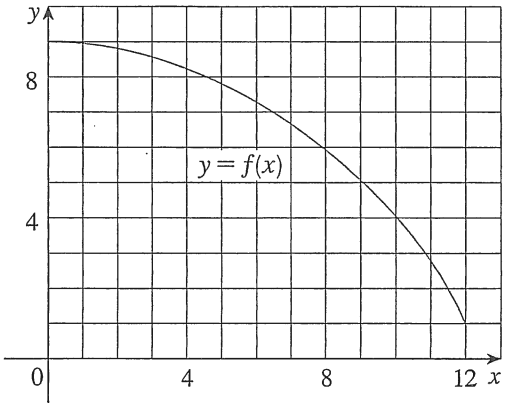
IB HL 2
Riemann Sums

1. (a) By reading values from the given graph of f, use five rectangles to find a lower estimate and an upper estimate for the area under the given graph of f from $x = 0$ to $x = 10$. In each case sketch the rectangles that you use.
(b) Find new estimates using ten rectangles in each case.



2. (a) Use six rectangles to find estimates of each type for the area under the given graph of f from $x = 0$ to $x = 12$.
(i) L_6 (sample points are left endpoints)
(ii) R_6 (sample points are right endpoints)

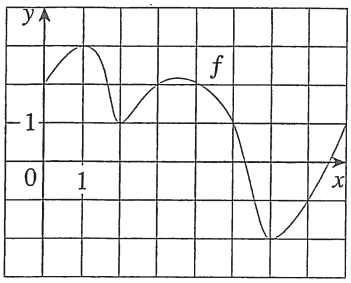
(b) Is L_6 an underestimate or overestimate of the true area?
(c) Is R_6 an underestimate or overestimate of the true area?



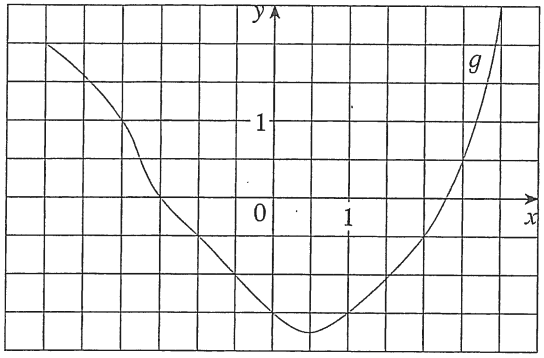
3. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (m/s)	0	1.9	3.3	4.5	5.5	5.9	6.2

4. The graph of a function f is given. Estimate $\int_0^8 f(x)\,dx$ using four subintervals with (a) right endpoints, (b) left endpoints.



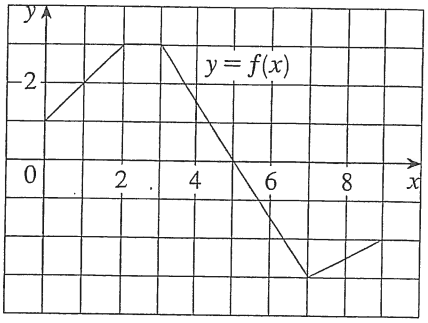
5. The graph of g is shown. Estimate $\int_{-3}^{3} g(x)\,dx$ with six subintervals using (a) right endpoints, (b) left endpoints



6. The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

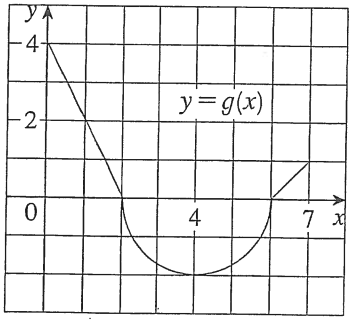
(a) $\int_0^2 f(x)\,dx$ (b) $\int_0^5 f(x)\,dx$

(c) $\int_5^7 f(x)\,dx$ (d) $\int_0^9 f(x)\,dx$



7. The graph of g consists of two straight lines and a semicircle. Use it to evaluate each integral.

(a) $\int_0^2 g(x)\,dx$ (b) $\int_2^6 g(x)\,dx$ (c) $\int_0^7 g(x)\,dx$



8. Evaluate the Riemann sum for $f(x) = 2 - x^2$, $0 \leq x \leq 2$, with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.