REVIEW SET 16C

1 For A(−1, 2, 3), B(2, 0, −1) and C(−3, 2, −4) find:

a the equation of the plane defined by A, B and C

b the measure of angle CAB

c r given that D(r, 1, $-r$) is a point such that angle BDC is a right angle.

2 **a** Find where the line through L(1, 0, 1) and M(−1, 2, −1) meets the plane with equation $x - 2y - 3z = 14$.

b Find the shortest distance from L to the plane.

3 Given A(−1, 2, 3), B(1, 0, −1) and C(1, 3, 0), find:

a the normal vector to the plane containing A, B and C

b D, the fourth vertex of parallelogram ACBD

c the coordinates of the foot of the perpendicular from C to the line AB.

4 Show that the line $x - 1 = \dfrac{y+2}{2} = \dfrac{z-3}{4}$ is parallel to the plane $6x + 7y - 5z = 8$ and find the distance between them.

5 Consider the lines with equations $\dfrac{x-3}{2} = y - 4 = \dfrac{z+1}{-2}$ and $x = -1 + 3t$, $y = 2 + 2t$, $z = 3 - t$.

a Are the lines parallel, intersecting or skew? Justify your answer.

b Determine the cosine of the acute angle between the lines.

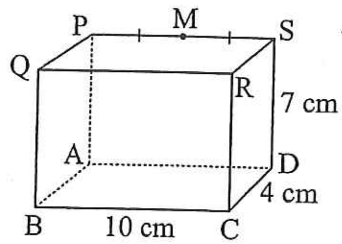
6 For A(2, −1, 3) and B(0, 1, −1), find:

a the vector equation of the line through A and B, and hence

b the coordinates of C on AB which is 2 units from A.

7 Find the equation of the plane through $A(-1, 2, 3)$, $B(1, 0, -1)$ and $C(0, -1, 5)$. If X is $(3, 2, 4)$, find the angle that AX makes with this plane.

8 **a** Find all vectors of length 3 units which are normal to the plane $x - y + z = 6$.
b Find a unit vector parallel to $\mathbf{i} + r\mathbf{j} + 3\mathbf{k}$ and perpendicular to $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.
c The distance from $A(-1, 2, 3)$ to the plane with equation $2x - y + 2z = k$ is 3 units. Find k.

9 

Use vector methods to determine the measure of angle QDM given that M is the midpoint of PS.

10 $P(-1, 2, 3)$ and $Q(4, 0, -1)$ are two points in space. Find:
a $\overrightarrow{PQ}$ **b** the angle that $\overrightarrow{PQ}$ makes with the X-axis.

11 ABC is a triangle in space. M is the midpoint of side [BC] and O is the origin. P is a point such that $\overrightarrow{OP} = \frac{1}{3}(\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$.
a Write $\overrightarrow{OM}$ in terms of $\overrightarrow{OB}$ and $\overrightarrow{OC}$.
b Hence, show that $\overrightarrow{OP} = \frac{1}{3}(\overrightarrow{OA} + 2\overrightarrow{OM})$.
c Show that P lies on [AM].
d Find the ratio in which P divides [AM].

12 Lines l_1 and l_2 are given by

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} + s\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad l_2: \quad \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

a Find the coordinates of A, the point of intersection of the lines.
b Show that the point $B(0, -3, 2)$ lies on the line l_2.
c Find the equation of the line BC given that $C(3, -2, -2)$ lies on l_1.
d Find the equation of the plane containing A, B and C.
e Find the area of triangle ABC.
f Show that the point $D(9, -4, 2)$ lies on the normal to the plane passing through C.
g Find the volume of the pyramid ABCD.