Quadratic Functions Review Paper 1 KEY

1. (a) $q = -2, r = 4$ or $q = 4, r = -2$ A1A1 N2

(b) $x = 1$ (must be an equation) A1 N1

(c) substituting (0, –4) into the equation (M1)
e.g. $-4 = p(0-(-2))(0-4)$, $-4 = p(-4)(2)$

correct working towards solution (A1)
e.g. $-4 = -8p$

$p = \frac{4}{8}\left(=\frac{1}{2}\right)$ A1 N2

[6]

2. (a) $p = -\frac{1}{2}$, $q = 2$ (A1)(A1) (C2)

or vice versa

(b) By symmetry C is midway between p, q (M1)

***Note:** This (M1) may be gained by implication.*

$\Rightarrow$ x-coordinate is $\frac{-\frac{1}{2}+2}{2} = \frac{3}{4}$ (A1) (C2)

[4]

3. (a) $2x^2 - 8x + 5 = 2(x^2 - 4x + 4) + 5 - 8$ (M1)
$= 2(x-2)^2 - 3$ (A1)(A1)(A1)
=> $a = 2$, $p = 2$, $q = -3$ (C4)

(b) Minimum value of $2(x-2)^2 = 0$ (or minimum value occurs when $x = 2$) (Ml)
$\Rightarrow$ Minimum value of $f(x) = -3$ (A1) (C2)
OR
Minimum value occurs at (2, –3) (M1)(A1) (C2)

[6]

4. (a) (i) $h = -1$ (A2) (C2)

(ii) $k = 2$ (A1) (C1)

(b) $a(1+1)^2 + 2 = 0$ (M1)(A1)
$a = -0.5$ (A1) (C3)

[6]

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5. $(7 - x)(1 + x) = 0$ (M1)

$\Leftrightarrow x = 7$ or $x = -1$ (A1)(C1)(C1)

$B: x = \dfrac{7 + -1}{2} = 3;$ (A1)

$y = (7 - 3)(1 + 3) = 16$ (A1) (C2)

[4]

6. Graph of quadratic function.

Expression	+	–	0
a			
c			
$b^2 - 4ac$			
b			

(A1) (C1)

(A1) (C1)

(A1) (C1)

(A1) (C1)

[4]

7. (a) $x^2 - 3x - 10 = (x - 5)(x + 2)$ (M1)(A1) (C2)

(b) $x^2 - 3x - 10 = 0 \Rightarrow (x - 5)(x + 2) = 0$ (M1)

$\Rightarrow x = 5$ or $x = -2$ (A1) (C2)

[4]

8. $4x^2 + 4kx + 9 = 0$

Only one solution $\Rightarrow b^2 - 4ac = 0$ (M1)

$16k^2 - 4(4)(9) = 0$ (A1)

$k^2 = 9$

$k = \pm 3$ (A1)

But given $k > 0$, $k = 3$ (A1) (C4)

OR

One solution $\Rightarrow (4x^2 + 4kx + 9)$ is a perfect square (M1)

$4x^2 + 4kx + 9 = (2x \pm 3)^2$ by inspection (A2)

given $k > 0$, $k = 3$ (A1) (C4)

[4]

9. (a) $f(x) = x^2 - 6x + 14$

$f(x) = x^2 - 6x + 9 - 9 + 14$ (M1)

$f(x) = (x - 3)^2 + 5$ (M1)

(b) Vertex is (3, 5) (A1)(A1)

[4]

10. (a) For a reasonable attempt to complete the square, (or expanding) (M1)

e.g. $3x^2 - 12x + 11 = 3(x^2 - 4x + 4) + 11 - 12$

$f(x) = 3(x - 2)^2 - 1$ (accept $h = 2$, $k = 1$) A1A1 N3

(b) **METHOD 1**

Vertex shifted to $(2 + 3, -1 + 5) = (5, 4)$ M1

so the new function is $3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

METHOD 2

$g(x) = 3((x - 3) - h)^2 + k + 5 = 3((x - 3) - 2)^2 - 1 + 5$ M1

$= 3(x - 5)^2 + 4$ (accept $p = 5$, $q = 4$) A1A1 N2

[6]

11. $y = (x + 2)(x - 3)$ (M1)

$= x^2 - x - 6$ (A1)

Therefore, $0 = 4 - 2p + q$ (A1)(A1)(C2)(C2)

OR

$y = x^2 - x - 6$ (C3)

OR

$0 = 4 - 2p + q$ (A1)

$0 = 9 + 3p + q$ (A1)

$p = -1$, $q = -6$ (A1)(A1)(C2)(C2)

[4]

12. (a) **METHOD 1**

Using the discriminant = 0 ($q^2 - 4(4)(25) = 0$) M1

$q^2 = 400$

$q = 20$, $q = -20$ A1A1 N2

METHOD 2

Using factorizing:

$(2x - 5)(2x - 5)$ and/or $(2x + 5)(2x + 5)$ M1

$q = 20$, $q = -20$ A1A1 N2

(b) $x = 2.5$ A1 N1

(c) $(0, 25)$ A1A1 N2

[6]

13. (a) Vertex is $(4, 8)$ A1A1 N2

(b) Substituting $-10 = a(7 - 4)^2 + 8$ M1

$a = -2$ A1 N1

(c) For y-intercept, $x = 0$ (A1)

$y = -24$ A1 N2

[6]

14. (a) For attempting to complete the square or expanding $y = 2(x - c)^2 + d$, or for showing the vertex is at (3, 5) M1

$y = 2(x - 3)^2 + 5$ (accept $c = 3$, $d = 5$) A1A1 N2

(b) (i) $k = 2$ A1 N1

(ii) $p = 3$ A1 N1

(iii) $q = 5$ A1 N1

[6]

15. (a) (i) $m = 3$ A2 N2

(ii) $p = 2$ A2 N2

(b) Appropriate substitution M1

eg $0 = d(1 - 3)^2 + 2$, $0 = d(5 - 3)^2 + 2$, $2 = d(3 - 1)(3 - 5)$

$d = -\frac{1}{2}$ A1 N1

[6]

16. (a) $a = 3$, $b = 4$ (A1)

$f(x) = (x - 3)^2 + 4$ A1 (C2)

(b) $y = (x - 3)^2 + 4$

METHOD 1

$x = (y - 3)^2 + 4$ (M1)

$x - 4 = (y - 3)^2$

$\sqrt{x - 4} = y - 3$ (M1)

$y = \sqrt{x - 4} + 3$ (A1) 3

METHOD 2

$y - 4 = (x - 3)^2$ (M1)

$\sqrt{y - 4} = x - 3$ (M1)

$\sqrt{y - 4} + 3 = x$

$y = \sqrt{x - 4} + 3$

$\Rightarrow f^{-1}(x) = \sqrt{x - 4} + 3$ (A1) 3

(c) $x \geq 4$ (A1)(C1)

[6]

17. One solution $\Rightarrow$ discriminant = 0 (M2)

$3^2 - 4k = 0$ (A2)

$9 = 4k$

$k = \frac{9}{4}\left(= 2\frac{1}{4}, 2.25\right)$ (A2) (C6)

Note: *If candidates correctly solve an incorrect equation, award M2 A0 A2(ft), if they have the first line or equivalent, otherwise award no marks.*

[6]

18. Discriminant $\Delta = b^2 - 4ac$ $(= (-2k)^2 - 4)$ (A1)

$\Delta > 0$ (M2)

Note: *Award (M1)(M0) for* $\Delta \geq 0$.

$(2k)^2 - 4 > 0 \Rightarrow 4k^2 - 4 > 0$

EITHER

$4k^2 > 4$ $(k^2 > 1)$ (A1)

OR

$4(k - 1)(k + 1) > 0$ (A1)

OR

$(2k - 2)(2k + 2) > 0$ (A1)

THEN

$k < -1$ or $k > 1$ (A1)(A1) (C6)

Note: *Award (A1) for* $-1 < k < 1$.

[6]

19. (a) $p = -1$ and $q = 3$ (or $p = 3$, $q = -1$) (A1)(A1) (C2)

(accept $(x + 1)(x - 3)$*)*

(b) **EITHER**

by symmetry (M1)

OR

differentiating $\frac{dy}{dx} = 2x - 2 = 0$ (M1)

OR

Completing the square (M1)

$x^2 + 2x - 3 = x^2 - 2x + 1 - 4 = (x - 1)^2 - 4$

THEN

$x = 1, y = -4$ (so C is $(1, -4)$) (A1)(A1)(C2)(C1)

(c) -3 (A1) (C1)

(accept $(0, -3)$*)*

[6]

20. (a) evidence of setting function to zero (M1)
e.g. $f(x) = 0$, $8x = 2x^2$

evidence of correct working A1

e.g. $0 = 2x(4 - x)$, $\frac{-8 \pm \sqrt{64}}{-4}$

x-intercepts are at 4 and 0 (accept (4, 0) and (0, 0), or $x = 4$, $x = 0$) A1A1 N1N1

(b) (i) $x = 2$ (must be equation) A1 N1

(ii) substituting $x = 2$ into $f(x)$ (M1)
$y = 8$ A1 N2

[7]

21. (a) Evidence of completing the square (M1)
$f(x) = 2(x^2 - 6x + 9) + 5 - 18$ (A1)
$= 2(x - 3)^2 - 13$ (accept $h = 3$, $k = 13$) A1 N3

(b) Vertex is (3, –13) A1A1 N2

(c) $x = 3$ (must be an equation) A1 N1

(d) evidence of using fact that $x = 0$ at y-intercept (M1)
y-intercept is (0, 5) (accept 5) A1 N2

(e) **METHOD 1**
evidence of using $y = 0$ at x-intercept (M1)
e.g. $2(x - 3)^2 - 13 = 0$
evidence of solving this equation (M1)
e.g. $(x - 3)^2 = \frac{13}{2}$ A1

$(x - 3) = \pm\sqrt{\frac{13}{2}}$

$x = 3 \pm \sqrt{\frac{13}{2}} = 3 \pm \frac{\sqrt{26}}{2}$ A1

$x = \frac{6 \pm \sqrt{26}}{2}$

$p = 6$, $q = 26$, $r = 2$ A1A1A1 N4

METHOD 2
evidence of using $y = 0$ at x-intercept (M1)
e.g. $2x^2 - 12x + 5 = 0$
evidence of using the quadratic formula (M1)

$x = \frac{12 \pm \sqrt{12^2 - 4 \times 2 \times 5}}{2 \times 2}$ A1

$x = \frac{12 \pm \sqrt{104}}{4}$ $\left(= \frac{6 \pm \sqrt{26}}{2}\right)$ A1

$p = 12$, $q = 104$, $r = 4$ (or $p = 6$, $q = 26$, $r = 2$) A1A1A1 N4

[15]

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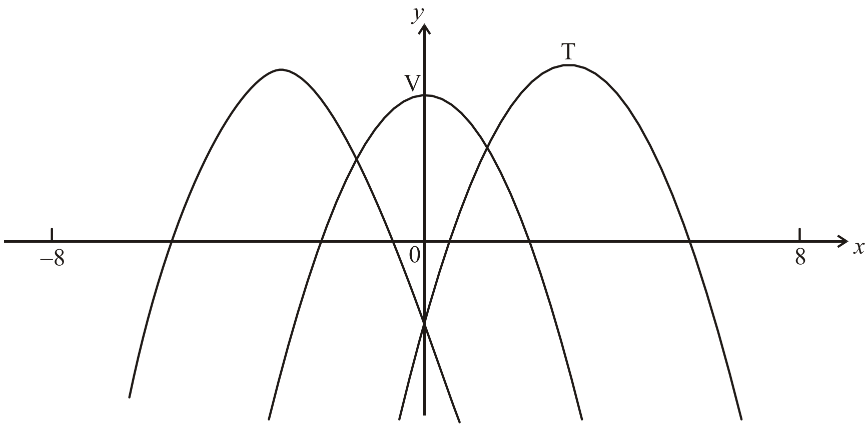
22. (a) (i) $h = 3$ A1 N1

(ii) $k = 1$ A1 N1

(b) $g(x) = f(x - 3) + 1$, $5 - (x - 3)^2 + 1$, $6 - (x - 3)^2$, $-x^2 + 6x - 3$ A2 N2

(c)



M1A1 N2

Note: *Award M1 for attempt to reflect through y-axis, A1 for vertex at approximately (– 3, 6).*

[6]

23. (a) $f(x) = -10(x + 4)(x - 6)$ A1A1 N2

(b) **METHOD 1**

attempting to find the x-coordinate of maximum point (M1)

e.g. averaging the x-intercepts, sketch, $y' = 0$, axis of symmetry

attempting to find the y-coordinate of maximum point (M1)

e.g. $k = -10(1+4)(1-6)$

$f(x) = -10(x-1)^2 + 250$ A1A1 N4 4

METHOD 2

attempt to expand $f(x)$ (M1)

e.g. $-10(x^2 - 2x - 24)$

attempt to complete the square (M1)

e.g. $-10((x-1)^2 - 1 - 24)$

$f(x) = -10(x-1)^2 + 250$ A1A1 N4 4

(c) attempt to simplify (M1)

e.g. distributive property, $-10(x-1)(x-1) + 250$

correct simplification A1

e.g. $-10(x^2 - 6x + 4x - 24)$, $-10(x^2 - 2x + 1) + 250$

$f(x) = 240 + 20x - 10x^2$ AG N0 2

(d) (i) valid approach (M1)

e.g. vertex of parabola, $v'(t) = 0$

$t = 1$ A1 N2

[10]