IBHL2 Test 1

Name ______________________

9/22/2016

Riemann Sums, Definite Integrals,
and the Fundamental Theorem of Calculus

Full marks are not necessarily awarded for a correct answer with no supporting work or explanations.

Evaluate, if possible.

(2) 1. $\int_0^1 \left(x^2+\sqrt{x}\right)dx$ $= \left(\frac{1}{3}x^3 + \frac{2}{3}x^{3/2}\right)\Big|_0^1 = \frac{1}{3} + \frac{2}{3} = \boxed{1}$

(1) (1)

(5) 2. $\int_0^{\pi/2} \frac{1+\cos 2x}{2}dx$ $= \int_0^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)dx$ (1)

(1) $u = 2x$, $du = 2\,dx$

$= \frac{1}{2}\int_0^{\pi} \frac{1}{2}du + \frac{1}{4}\int_0^{\pi}\cos u\,du$ (1)

(1) $= \frac{1}{4}u\Big|_0^{\pi} + \frac{1}{4}\sin u\Big|_0^{\pi} = \frac{\pi}{4} + 0 = \boxed{\frac{\pi}{4}}$ (1)

(4) 3. $\int_0^{\sqrt{7}} x\left(x^2+1\right)^{1/3}dx$

$u = x^2+1$
$du = 2x\,dx$
(1)

$\frac{1}{2}\int_1^8 u^{1/3}du = \frac{1}{2}\cdot\frac{3}{4}u^{4/3}\Big|_1^8 = \frac{3}{8}(16-1) = \boxed{\frac{45}{8}}$

(1) (1) (1)

(4) 4. $\int_0^{\pi/6} \cos^{-3}2\theta\sin 2\theta\, d\theta$

(1) $u = \cos 2\theta$
$du = -2\sin 2\theta\, d\theta$

$-\frac{1}{2}\int_1^{1/2} u^{-3}du = \frac{1}{2}\int_{1/2}^{1} u^{-3}du$

(1)

$= \frac{1}{2}\cdot\frac{-1}{2}u^{-2}\Big|_{1/2}^{1}$ (1)

$= -\frac{1}{4}(1-4) = -\frac{1}{4}+1 = \boxed{\frac{3}{4}}$

(1)

(2) 5. $\frac{d}{dx}\int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\sqrt{1-\sin^2 x}} \cdot \cos x = \frac{\cos x}{\cos x} = \boxed{1}$

(1) (1)

(3) 6. If f is continuous and $\int_0^2 f(x)dx = 6$, evaluate $\int_0^{\pi/2} f(2\sin\theta)\cos\theta d\theta$.

$u = 2\sin\theta$
$du = 2\cos\theta d\theta$ (1)

$\frac{1}{2}\int_0^2 f(u)\,du = \boxed{3}$

(1) (1)

(4) 7. If f' is continuous on $[a,b]$, show that $2\int_a^b f(x)f'(x)dx = [f(b)]^2 - [f(a)]^2$.

(1) $u = f(x)$
$du = f'(x)dx$

(1) $2\int_{f(a)}^{f(b)} u\,du = 2\cdot\frac{1}{2}u^2 \Big|_{f(a)}^{f(b)}$

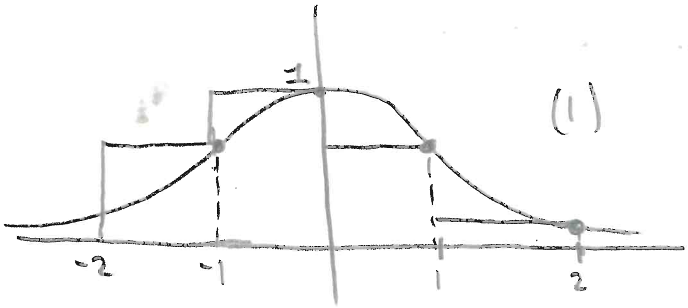
(1)

$= [f(b)]^2 - [f(a)]^2$

(1)

8.

(2)



(1)

$$\frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{5} = \boxed{\frac{11}{5}}$$

(1)