

Prove the following statement using mathematical induction:

1. $\sum_{i=1}^n (2i-1) = n^2, \forall n \in \mathbb{Z}^+$

2. $a_n = \frac{3^n - 1}{2}$ is the explicit formula for the sequence $\begin{cases} a_1 = 1 \\ a_{k+1} = 3a_k + 1 \end{cases}, \forall n \in \mathbb{Z}^+$

3. $\sum_{i=2}^n \frac{1}{i(i-1)} = \frac{n-1}{n}$, for all integers $n, n > 1$

4. 12 is a factor of $13^n - 1, \forall n \in \mathbb{Z}^+$

5. 6 is a factor of $n^3 + 11n \forall n \in \mathbb{Z}^+$

6. Prove that the sum of the geometric series $1 + r + r^2 + r^3 + \dots + r^{n-1} = \frac{1-r^n}{1-r}$ for all integers $n \geq 1$.