(A) Lesson Context

BIG PICTURE of this UNIT:	• What is a Polynomial and how do they look? • What are the attributes of a Polynomial? • How do I work with Polynomials?		
CONTEXT of this LESSON:	Where we've been We have discussed the basic appearance of graphs of polynomial functions and algebraic processes associated with factoring polynomials	Where we are How can we use polynomials to model real world scenarios?	Where we are heading How do we work with polynomial models?

(B) Lesson Objectives:

a. Model real world scenarios using polynomial functions & the TI-84.
b. Introduce the concept of average and instantaneous rates of change using polynomial functions.

(C) Example 1

SureGrip Athletic shoes tracks the relationship between total sales of shoes and the advertising expenses. The function used to model the relationship is $S(d) = -\frac{1}{4{,}000}d^3 + \frac{3}{20}d^2$, where *S* is shoes sales in millions of dollars and ***d*** is measured in tens of thousands of dollars.

1. Explain what the ordered pair $S(200) = 4000$ means in the context of this model.
2. Graph the function, given that the domain for context of this problem is restricted to $\{d \in R \mid 50 \le d \le 500\}$.
3. What advertising **expense (cost)** would optimize the sale of shoes? What would be the optimal sales of shoes (revenues)?
4. If the company wants to have sales of 6.5 billion dollars (S = 6500), how much should they spend on advertising expenses?

(D) Example 2

The volume of a box is modeled by the equation $V(x) = x^3 - 15x^2 + 66x - 80$. Graph the function in a standard view window.

1. Re-adjust the window to a more reasonable setting, given what the graph looks like in the standard view window.
2. Factor the expression $x^3 - 15x^2 + 66x - 80$ and HENCE determine expressions for the dimensions of the box.
3. Evaluate and interpret $V(3.5)$.
4. Determine the optimal volume of the box. What are the dimensions that optimize the volume of the box?
5. Explain why x = 7 is inadmissible in the context of this question?
6. Suggest a domain for this model and explain/justify you domain.

(E) Example 3

The table included shows the population of Thunder Bay from 1966 to 1998. Using x as the number of years since 1966, determine a quartic model for this data set.

Year	1966	1976	1986	1996	1998
population	143,673	119,253	122,217	125,562	128,607

1. Write the equation as P(x) = where each coefficient will be rounded to two decimal places.
2. Explain why Mr. S has decide to restrict the domain for this model to $\{x \in R \mid 0 \leq x \leq 32\}$.
3. Determine when the population was 122,775.
4. When was the population of Thunder Bay at a minimum? What was this minimum population.

NEW CONCEPT: RATES OF CHANGE

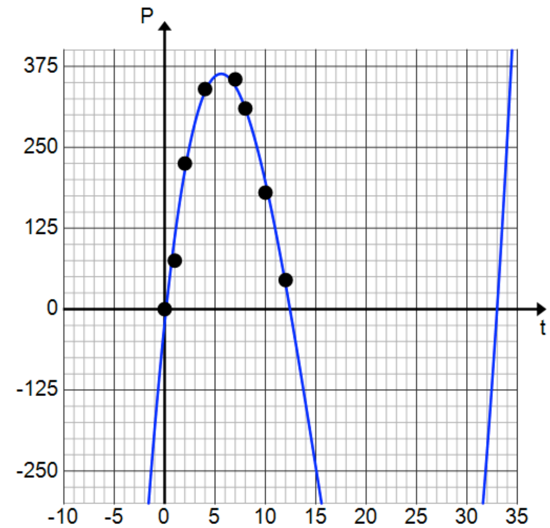
5. Use the internet to define and illustrate these terms: (i) secant line & (ii) tangent line
6. Determine the value of P(0) as well as P(8)
7. Evaluate and interpret the DIFFERENCE QUOTIENT $\frac{P(8)-P(0)}{8-0}$ and explain what the quotient means in context.
   a. Now repeat for the DIFFERENCE QUOTIENT $\frac{P(8)-P(5)}{8-5}$.
   b. Now repeat for the DIFFERENCE QUOTIENT $\frac{P(8)-P(7)}{8-7}$.
   c. Now repeat for the DIFFERENCE QUOTIENT $\frac{P(8)-P(7.9)}{8-7.9}$.
   d. Now repeat for the DIFFERENCE QUOTIENT $\frac{P(8)-P(7.99)}{8-7.99}$.
   e. Use the TI-84 to draw the tangent line at x = 8
   f. Explain what is happening mathematically & contextually.
   g. HL Extension ➔ Explain what the statement $\lim_{x \to a} \frac{P(x)-P(a)}{x-a}$ means.

Mathematical Modeling Exercise

The owners of Dizzy Lizzy's, an amusement park, are studying the wait time at their most popular roller coaster. The table below shows the number of people standing in line for the roller coaster t hours after Dizzy Lizzy's opens.

t (hours)	0	1	2	4	7	8	10	12
P (people in line)	0	75	225	345	355	310	180	45

Jaylon made a scatterplot and decided that a cubic function should be used to model the data. His scatterplot and curve are shown below.



1. Do you agree that a cubic polynomial function is a good model for this data? Explain.

2. What information would Dizzy Lizzy's be interested in learning about from this graph? How could they determine the answer?

3. Estimate the time at which the line is the longest. Explain how you know.

4. Estimate the number of people in line at that time. Explain how you know.

5. Estimate the t-intercepts of the function used to model this data.

6. Use the t-intercepts to write a formula for the function of the number of people in line, f, after t hours.

7. Use the relative maximum to find the leading coefficient of f. Explain your reasoning.

8. What would be a reasonable domain for your function f? Why?

9. Use your function f to calculate the number of people in line 10 hours after the park opens.

10. Comparing the value calculated above to the actual value in the table, is your function f an accurate model for the data? Explain.

11. Use the regression feature of a graphing calculator to find a cubic function g to model the data.

12. Graph the function f you found and the function g produced by the graphing calculator. Use the graphing calculator to complete the table. Round your answers to the nearest integer.

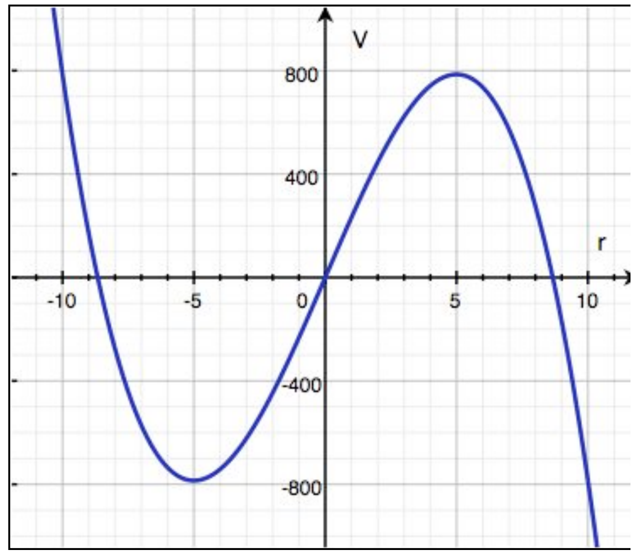
***t* (hours)**	*0*	*1*	*2*	*4*	*7*	*8*	*10*	*12*
***P* (people in line)**	*0*	*75*	*225*	*345*	*355*	*310*	*180*	*45*
***f(t)* (your equation)**								
***g(t)* (regression eqn.)**								

13. Based on the results from the table, which model was more accurate at *t=2* hours? *t=10* hours?

Name ____ Date ______________

Lesson 4.6: Modeling with Polynomials

Exit Ticket



Jeannie wishes to construct a cylinder closed at both ends.
The figure at right shows the graph of a cubic polynomial function, V, used to model the volume of the cylinder as a function of the radius if the cylinder is constructed using $150\pi\ cm^3$ of material. Use the graph to answer the questions below. Estimate values to the nearest half unit on the horizontal axis and to the nearest *50* units on the vertical axis.

What are the zeros of the function V?

What is the relative maximum value of V, and where does it occur?

The equation of this function is $V(r) = c(r^3 - 72.25r)$ for some real number c. Find the value of c so that this formula fits the graph.

Use the graph to estimate the volume of the cylinder with $r=2$ cm.

Use your formula for V to find the volume of the cylinder when $r=2$ cm. How close is the value from the formula to the value on the graph?