

Homogeneous Differential Equations

By using the substitution $y = vx$,

1. show that the general solution to $x \frac{dy}{dx} = x + y$ can be written as $y = x \ln|Ax|$
2. show that the general solution to $\frac{dy}{dx} = \frac{y(x-y)}{x^2}$ can be written as $y = \frac{x}{\ln|Ax|}$
3. show that the general solution to $x^2 \frac{dy}{dx} = 3x^2 + xy$ can be written as $y = x \ln|Ax^3|$
4. show that the general solution to $2xy \frac{dy}{dx} = x^2 + 3y^2$ can be written as $y^2 = x^2(|Ax| - 1)$
5. show that the general solution to $x^2 \frac{dy}{dx} = x^2 + xy + y^2$ can be written as $y = x \tan(\ln|Ax|)$
6. show that the general solution to $(x^2 + y^2) \frac{dy}{dx} = xy$ can be written as $e^{\frac{x^2}{y^2}} = Ay^2$
7. find the particular solution to Exercise 2 if $y(1) = 1$
8. find the particular solution to Exercise 5 if $y(3) = \sqrt{3}$