Exponentials and Logarithms Review Paper 2 KEY

1. (a) combining 2 terms (A1)

e.g. $\log_3 8x - \log_3 4$, $\log_3 \frac{1}{2}x + \log_3 4$

expression which clearly leads to answer given A1

e.g. $\log_3 \frac{8x}{3}$, $\log_3 \frac{4x}{2}$

$f(x) = \log_3 2x$ AG N0 2

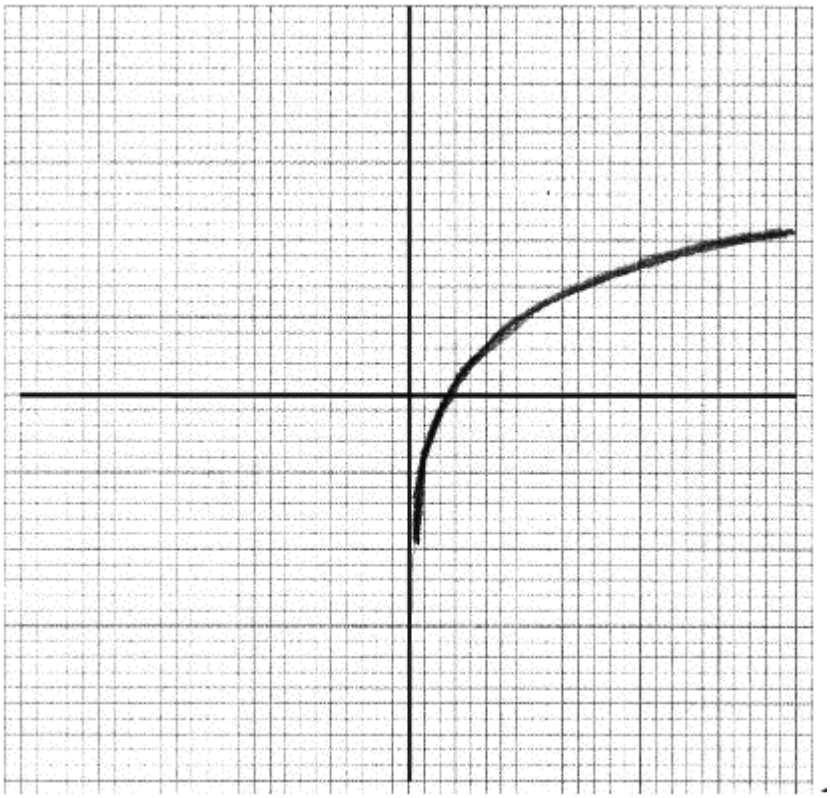
(b) attempt to substitute either value into f (M1)

e.g. $\log_3 1$, $\log_3 9$

$f(0.5) = 0, f(4.5) = 2$ A1A1 N3 3

(c) (i) $a = 2, b = 3$ A1A1 N1N1

(ii)



A1A1A1 N3

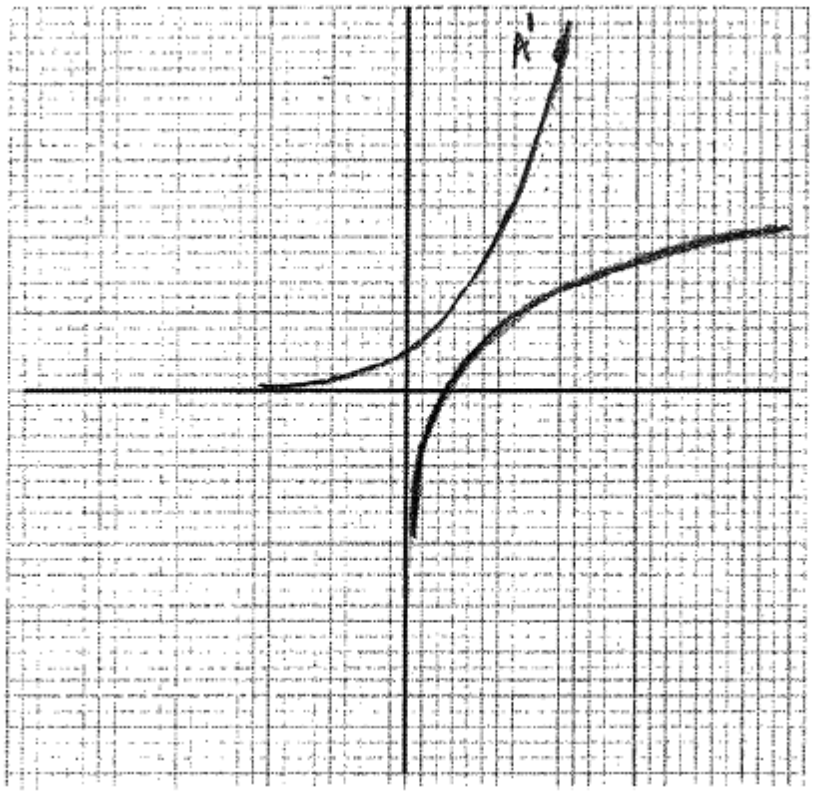
Note: *Award A1 for sketch approximately through (0.5 ± 0.1, 0 ± 0.1)*
A1 for approximately correct shape,
A1 for sketch asymptotic to the y-axis.

(iii) $x = 0$ (must be an equation) A1 N1

[6]

(d) $f^{-1}(0) = 0.5$ A1 N1 1

(e)



A1A1A1A1 N4 4

Note: *Award A1 for sketch approximately through (0 ± 0.1, 0.5 ± 0.1),*
A1 for approximately correct shape of the graph reflected over y = x,
A1 for sketch asymptotic to x-axis,
*A1 for point (2 ± 0.1, 4.5 ± 0.1) clearly marked **and** on curve.*

[16]

Exponentials and Logarithms Review Paper 2 KEY

2. (a) $n = 800e^0$ (A1)
$n = 800$ A1 N2

(b) evidence of using the derivative (M1)
$n'(15) = 731$ A1 N2

(c) **METHOD 1**

setting up inequality (accept equation or reverse inequality) A1
e.g. $n'(t) > 10\ 000$

evidence of appropriate approach M1
e.g. sketch, finding derivative

$k = 35.1226...$ (A1)
least value of k is 36 A1 N2

METHOD 2

$n'(35) = 9842$, **and** $n'(36) = 11208$ A2
least value of k is 36 A2 N2

[8]

Exponentials and Logarithms Review Paper 2 KEY

3. (a) (i) $n = 5$ (A1)

$T = 280 \times 1.12^5$

$T = 493$ A1 N2

(ii) evidence of doubling (A1)
e.g. 560
setting up equation A1
e.g. $280 \times 1.12^n = 560$, $1.12^n = 2$
$n = 6.116...$ (A1)
in the year 2007 A1 N3

(b) (i) $P = \dfrac{2\,560\,000}{10 + 90\,e^{-0.1(5)}}$ (A1)

$P = 39\,635.993...$ (A1)

$P = 39\,636$ A1 N3

(ii) $P = \dfrac{2\,560\,000}{10 + 90\,e^{-0.1(7)}}$

$P = 46\,806.997...$ A1
not doubled A1 N0
valid reason for **their** answer R1
e.g. $P < 51200$

(c) (i) correct value A2 N2

$e.g. \dfrac{25600}{280}, 91.4, 640:7$

(ii) setting up an inequality (accept an equation, or reversed inequality) M1

$e.g. \dfrac{P}{T} < 70, \dfrac{2\,560\,000}{\left(10 + 90e^{-0.1n}\right)280 \times 1.12^n} < 70$

finding the value 9.31.... (A1)

after 10 years A1 N2

[17]

4. (a) At $t = 2$, $N = 10e^{0.4(2)}$ (M1)
$N = 22.3$ (3 sf)
Number of leopards = 22 (A1)

(b) If $N = 100$, then solve $100 = 100e^{0.4t}$
$10 = e04^t$
$\ln 10 = 0.4t$
$t = \dfrac{\ln 10}{0.4} \sim 5.76$ years (3 sf) (A1)

[4]

Exponentials and Logarithms Review Paper 2 KEY

5. (a) $\frac{15.2}{1.027} = 14.8$ million (M1)(A1) (C2)

(b) $\frac{15.2}{(1.027)^5} = 13.3$ million (M1)(A1) (C2)

OR

$\frac{14.8}{(1.027)^4} = 13.3$ million (M1)(A1) (C2)

[4]

Exponentials and Logarithms Review Paper 2 KEY

Problems from Paper 1 that are really Paper 2…

3. (a) 253250 (accept 253000) A1 N1

(b) 1972 → 2002 is 30 years, increase of 1.3% → 1.013 (A1)(A1)

Evidence of any appropriate approach (M1)

Correct substitution 250000×1.013^{30} A1

368000 (accept 368318) A1 N3

[6]

13. $10\,000e^{-0.3t} = 1500$ (A1)

For taking logarithms (M1)

$-0.3t \ln e = \ln 0.15$ (A1)

$t = \dfrac{\ln 0.15}{-0.3}$ (A1)

$= 6.32$ (A1)

7 (years) (A1) (C6)

***Note**: Candidates may use a graphical method. Award (A1) for setting up the correct equation, (M1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 6.32, and (A1) for 7.*

[6]

15. (a) $p = 100e^0$ (M1)

$= 100$ (A1) (C2)

(b) $300=100e^{0.05t}$ (M1)

Correct reasoning (sketch of graph) (A1)

t = 21.97, 22 hours (A1) (A1) (C4)

[6]

16. (a) Initial mass $\Rightarrow t = 0$ (A1)

mass = 4 (A1) (C2)

(b) $1.5 = 4e^{-0.2t}$ (or $0.375 = e^{-0.2t}$) (M2)

$\ln 0.375 = -0.2t$ (M1)

$t = 4.90$ hours (A1) (C4)

[6]

17. $1.023^t = 2$ (M1)

$\Rightarrow t = \dfrac{\ln 2}{\ln 1.023}$ (M1)(A1)

$= 30.48...$

30 minutes (nearest minute) (A1) (C4)

***Note:** Do not accept 31 minutes.*

[4]