

## Exponentials and Logarithms Review Paper 2

1. Let  $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$ , for  $x > 0$ .

(a) Show that  $f(x) = \log_3 2x$ .

(2)

(b) Find the value of  $f(0.5)$  and of  $f(4.5)$ .

(3)

The function  $f$  can also be written in the form  $f(x) = \frac{\ln ax}{\ln b}$ .

(c) (i) Write down the value of  $a$  and of  $b$ .

(ii) Hence on graph paper, **sketch** the graph of  $f$ , for  $-5 \leq x \leq 5$ ,  $-5 \leq y \leq 5$ , using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote.

(6)

(d) Write down the value of  $f^{-1}(0)$ .

(1)

The point A lies on the graph of  $f$ . At A,  $x = 4.5$ .

(e) On your diagram, sketch the graph of  $f^{-1}$ , noting clearly the image of point A.

(4)

(Total 16 marks)

2. The number of bacteria,  $n$ , in a dish, after  $t$  minutes is given by  $n = 800e^{0.13t}$ .

(a) Find the value of  $n$  when  $t = 0$ .

(2)

(b) Find the rate at which  $n$  is increasing when  $t = 15$ .

(2)

(c) After  $k$  minutes, the rate of increase in  $n$  is greater than 10 000 bacteria per minute. Find the least value of  $k$ , where  $k \in \mathbb{Z}$ .

(4)

(Total 8 marks)

## Exponentials and Logarithms Review Paper 2

3. A city is concerned about pollution, and decides to look at the number of people using taxis. At the end of the year 2000, there were 280 taxis in the city. After  $n$  years the number of taxis,  $T$ , in the city is given by

$$T = 280 \times 1.12^n.$$

- (a) (i) Find the number of taxis in the city at the end of 2005.  
(ii) Find the year in which the number of taxis is double the number of taxis there were at the end of 2000.

(6)

- (b) At the end of 2000 there were 2 560 000 people in the city who used taxis. After  $n$  years the number of people,  $P$ , in the city who used taxis is given by

$$P = \frac{2560000}{10 + 90e^{-0.1n}}.$$

- (i) Find the value of  $P$  at the end of 2005, giving your answer to the nearest whole number.  
(ii) After seven complete years, will the value of  $P$  be double its value at the end of 2000? Justify your answer.

(6)

- (c) Let  $R$  be the ratio of the number of people using taxis in the city to the number of taxis. The city will reduce the number of taxis if  $R < 70$ .

- (i) Find the value of  $R$  at the end of 2000.  
(ii) After how many complete years will the city first reduce the number of taxis?

(5)

(Total 17 marks)

## Exponentials and Logarithms Review Paper 2

4. A group of ten leopards is introduced into a game park. After  $t$  years the number of leopards,  $N$ , is modeled by  $N = 10 e^{0.4t}$ .
- (a) How many leopards are there after 2 years?
  - (b) How long will it take for the number of leopards to reach 100? Give your answers to an appropriate degree of accuracy.

Give your answers to an appropriate degree of accuracy.

*Working:*

*Answers:*

- (a) .....
- (b) .....

**(Total 4 marks)**

5. Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.
- (a) What was the population one year ago?
  - (b) What was the population five years ago?

*Working:*

*Answers:*

- (a) .....
- (b) .....

**(Total 4 marks)**

## Exponentials and Logarithms Review Paper 2

### Problems from Paper 1 that are really Paper 2...

3. The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973.

(b) Find the population at the end of 2002.

(Total 6 marks)

13. A machine was purchased for \$10000. Its value  $V$  after  $t$  years is given by  $V = 10000e^{-0.3t}$ . The machine must be replaced at the end of the year in which its value drops below \$1500. Determine in how many years the machine will need to be replaced.

*Working:*

*Answers:*

(Total 6 marks)

15. The population  $p$  of bacteria at time  $t$  is given by  $p = 100e^{0.05t}$ .

Calculate

(a) the value of  $p$  when  $t = 0$ ;

(b) the rate of increase of the population when  $t = 10$ .

*Working:*

*Answers:*

(a) .....

(b) .....

(Total 6 marks)

## Exponentials and Logarithms Review Paper 2

16. The mass  $m$  kg of a radio-active substance at time  $t$  hours is given by

$$m = 4e^{-0.2t}.$$

- (a) Write down the initial mass.
- (b) The mass is reduced to 1.5 kg. How long does this take?

*Working:*

*Answers:*

(a) .....

(b) .....

**(Total 6 marks)**

17. A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

*Working:*

*Answer:*

.....

**(Total 4 marks)**