Exponentials and Logarithms Review Paper 1 KEY

1. (a) For finding second, third and fourth terms correctly (A1)(A1)(A1)

Second term $\binom{4}{1}e^3\left(\frac{1}{e}\right)$, third term $\binom{4}{1}e^2\left(\frac{1}{e}\right)^2$,

fourth term $\binom{4}{1}e\left(\frac{1}{e}\right)^3$

For finding first and last terms, **and** adding them to **their** three terms (A1)

$$\left(e+\frac{1}{e}\right)^4=\binom{4}{0}e^4+\binom{4}{1}e^3\left(\frac{1}{e}\right)+\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+\binom{4}{3}e\left(\frac{1}{e}\right)^3+\binom{4}{4}\left(\frac{1}{e}\right)^4$$

$$\left(e+\frac{1}{e}\right)^4=e^4+4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2+4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4+4e^2+6+\frac{4}{e^2}+\frac{1}{e^4}\right)$$ N4

(b) $$\left(e-\frac{1}{e}\right)^4=e^4-4e^3\left(\frac{1}{e}\right)+6e^2\left(\frac{1}{e}\right)^2-4e\left(\frac{1}{e}\right)^3+\left(\frac{1}{e}\right)^4$$

$$\left(=e^4-4e^2+6-\frac{4}{e^2}+\frac{1}{e^4}\right)$$ (A1)

Adding gives $2e^4 + 12 + \frac{2}{e^4}$

$$\left(\text{accept } 2\binom{4}{0}e^4+2\binom{4}{2}e^2\left(\frac{1}{e}\right)^2+2\binom{4}{4}\left(\frac{1}{e}\right)^4\right)$$ A1 N2

[6]

2. (a) $f^{-1}(x)=\ln x$ A1 N1

(b) (i) Attempt to form composite $(f \circ g)(x) = f(\ln(1 + 2x))$ (M1)

$(f \circ g)(x) = e^{\ln(1+2x)} = (= 1 + 2x)$ A1 N2

(ii) Simplifying $y = e^{\ln(1+2x)}$ to $y = 1 + 2x$ (may be seen in part (i) or later) (A1)

Interchanging x and y (may happen any time) M1

eg $x = 1 + 2y$ $x - 1 = 2y$

$(f \circ g)^{-1}(x) = \frac{x-1}{2}$ A1 N2

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4. METHOD 1

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - \log_{10} y^2 - \log_{10}\sqrt{z}$ (A1)(A1)(A1)

$\log_{10} y^2 = 2\log_{10} y$ (A1)

$\log_{10}\sqrt{z} = \frac{1}{2}\log z$ (A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10} x - 2\log y - \frac{1}{2}\log z$

$= p - 2q - \frac{1}{2}r$ (A1) (C2)(C2)(C2)

METHOD 2

$x = 10, y^2 = 10^{2p}, \sqrt{z} = 10^{\frac{r}{2}}$ (A1)(A1)(A1)

$\log_{10}\left(\frac{x}{y^2\sqrt{z}}\right) = \log_{10}\left(\frac{10^p}{10^{2q}10^{\frac{r}{2}}}\right)$ (A1)

$= \log_{10}\left(10^{p-2q-\frac{r}{2}}\right)\left(= p - 2q - \frac{r}{2}\right)$ (A2) (C2)(C2)(C2)

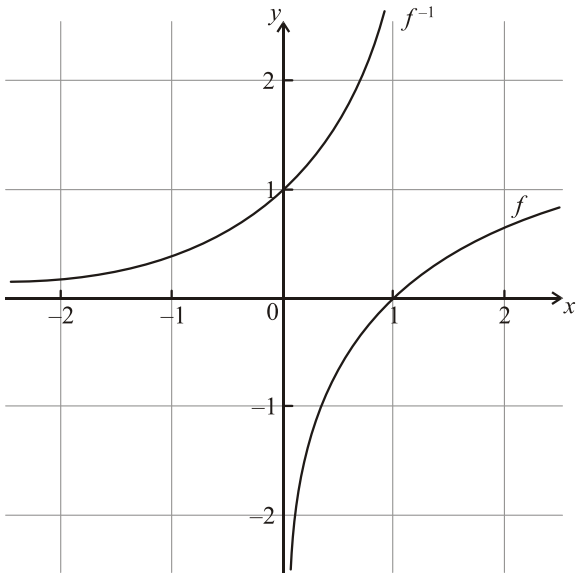
[6]

5. (a) (i) $f(a) = 1$ A1 N1

(ii) $f(1) = 0$ A1 N1

(iii) $f(a^4) = 4$ A1 N1

(b)



A1A1A1 N3

***Note**: Award A1 for approximate reflection of f in y = x, A1 for y intercept at 1, and A1 for curve asymptotic to x axis.*

[6]

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6. (a) $\log_5 x^2 = 2 \log_5 x$ (M1)

$= 2y$ (A1) (C2)

(b) $\log_5 \frac{1}{x} = -\log_5 x$ (M1)

$= -y$ (A1) (C2)

(c) $\log_{25} x = \frac{\log_5 x}{\log_5 25}$ (M1)

$= \frac{1}{2} y$ (A1) (C2)

[6]

7. (a) $\log_3 x - \log_3 (x-5) = \log_3 \left(\frac{x}{x-5} \right)$ (A1)

$A = \frac{x}{x-5}$ (A1) (C2)

Note: *If candidates have an incorrect or no answer to part (a) award (A1)(A0)*

if $\log \left(\frac{x}{x-5} \right)$ *seen in part (b).*

(b) **EITHER**

$\log_3 \left(\frac{x}{x-5} \right) = 1$

$\frac{x}{x-5} = 3^1 \left(= 3 \right)$ (M1)(A1)(A1)

$x = 3x - 15$

$-2x = -15$

$x = \frac{15}{2}$ (A1) (C4)

OR

$\frac{\log_{10} \left(\frac{x}{x-5} \right)}{\log_{10} 3} = 1$ (M1)(A1)

$\log_{10} \left(\frac{x}{x-5} \right) = \log_{10} 3$ (A1)

$x = 7.5$ (A1) (C4)

[6]

8. (a) 5 A1 N1

(b) **METHOD 1**

$\log_2\left(\frac{32^x}{8^y}\right) = \log_2 32^x - \log_2 8^y$ (A1)

$= x \log_2 32 - y \log_2 8$ (A1)

$\log_2 8 = 3$ (A1)

$p = 5$, $q = -3$ (accept $5x - 3y$) A1 N3

METHOD 2

$\frac{32^x}{8^y} = \frac{(2^5)^x}{(2^3)^y}$ (A1)

$= \frac{2^{5x}}{2^{3y}}$ (A1)

$= 2^{5x-3y}$ (A1)

$\log_2 (2^{5x-3y}) = 5x - 3y$
$p = 5$, $q = -3$ (accept $5x - 3y$) A1 N3

[5]

9. $\log_{10}\left(\frac{P}{QR^3}\right)^2 = 2\log_{10}\left(\frac{P}{QR^3}\right)$ (M1)

$2\log_{10}\left(\frac{P}{QR^3}\right) = 2(\log_{10}P - \log_{10}(QR^3))$ (M1)

$= 2(1\log_{10}P - \log_{10}Q - 3\log_{10}R)$ (M1)
$= 2(x - y - 3z)$
$= 2x - 2y - 6z$ or $2(x - y - 3z)$ (A1)

[4]

10. (a) $e^{\ln(x+2)} = e^3$ (M1)

$x + 2 = e^3$ (A1)

$x = e^3 - 2$ (= 18.1) A1 N3

(b) $\log_{10} (10^{2x}) = \log_{10} 500$ (accept lg and log for $\log_{10}$) (M1)

$2x = \log_{10} 500$ (A1)

$x = \frac{1}{2} \log_{10} 500 \quad \left(= \frac{\log 500}{\log 100} = 1.35\right)$ A1 N3

***Note**: In both parts (a) and (b), if candidates use a graphical approach, award **M1** for a sketch, A1 for indicating appropriate points of intersection, and A1 for the answer.*

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11. METHOD 1

$\log x^2 = 2\log x$ (A1)

$\log\sqrt{y} = \frac{1}{2}\log y$ (A1)

$\log z^3 = 3\log z$ (A1)

$2\log x + \frac{1}{2}\log y - 3\log z$ (A1)(A1)

$2a + \frac{1}{2}b - 3c$ (A1) (C6)

METHOD 2

$x^2 = 10^{2a},\ \sqrt{y} = 10^{\frac{b}{2}},\ z^3 = 10^{3c}$ (A1)(A1)(A1)

$\log_{10}\left(\frac{x^2\sqrt{y}}{z^3}\right) = \log_{10}\left(\frac{10^{2a}\times 10^{\frac{b}{2}}}{10^{3c}}\right)$ (A1)

$= \log_{10}\left(10^{2a+\frac{b}{2}-3c}\right)\ \left(= 2a + \frac{b}{2} - 3c\right)$ (A2)

[6]

12. METHOD 1

$9 = 3^2,\ 27 = 3^3$ (A1)(A1)

expressing as a power of 3, $(3^2)^{2x} = (3^3)^{1-x}$ (M1)

$3^{4x} = 3^{3-3x}$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

METHOD 2

$2x\log 9 = (1-x)\log 27$ (M1)(A1)(A1)

$\frac{2x}{1-x} = \frac{\log 27}{\log 9}\left(= \frac{3}{2}\right)$ (A1)

$4x = 3 - 3x$ (A1)

$7x = 3$

$\Rightarrow x = \frac{3}{7}$ (A1) (C6)

Notes: *Candidates may use a graphical method.*
Award (M1)(A1)(A1) for a sketch, (A1) for showing the point of intersection, (A1) for 0.4285…., and (A1) for $\frac{3}{7}$.

[6]

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14. (a) (i) $\log_c 15 = \log_c 3 + \log_c 5$ (A1)

$= p + q$ A1 N2

(ii) $\log_c 25 = 2 \log_c 5$ (A1)

$= 2q$ A1 N2

(b) **METHOD 1**

$d^{\frac{1}{2}} = 6$ M1

$d = 36$ A1 N1

METHOD 2

For changing base M1

eg $\frac{\log_{10} 6}{\log_{10} d} = \frac{1}{2}, 2\log_{10} 6 = \log_{10} d$

$d = 36$ A1 N1

[6]

18. (a) $\ln a^3 b = 3\ln a + \ln b$ (A1)(A1)

$\ln a^3 b = 3p + q$ A1 N3

(b) $\ln \frac{\sqrt{a}}{b} = \frac{1}{2} \ln a - \ln b$ (A1)(A1)

$\ln \frac{\sqrt{a}}{b} = \frac{1}{2} p - q$ A1 N3

[6]