

Exponentials and Logarithms Review Paper 1

1. (a) Expand $\left(e + \frac{1}{e}\right)^4$ in terms of e .

(4)

(b) Express $\left(e + \frac{1}{e}\right)^4 + \left(e - \frac{1}{e}\right)^4$ as the sum of three terms.

(2)

(Total 6 marks)

2. The functions $f(x)$ and $g(x)$ are defined by $f(x) = e^x$ and $g(x) = \ln(1 + 2x)$.

(a) Write down $f^{-1}(x)$.

(b) (i) Find $(f \circ g)(x)$.

(ii) Find $(f \circ g)^{-1}(x)$.

(Total 6 marks)

4. Let $p = \log_{10} x$, $q = \log_{10} y$ and $r = \log_{10} z$.

Write the expression $\log_{10} \left(\frac{x}{y^2 \sqrt{z}} \right)$ in terms of p , q and r .

Working:

Answer:

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(Total 6 marks)

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5. Let $f(x) = \log_a x, x > 0$.

(a) Write down the value of

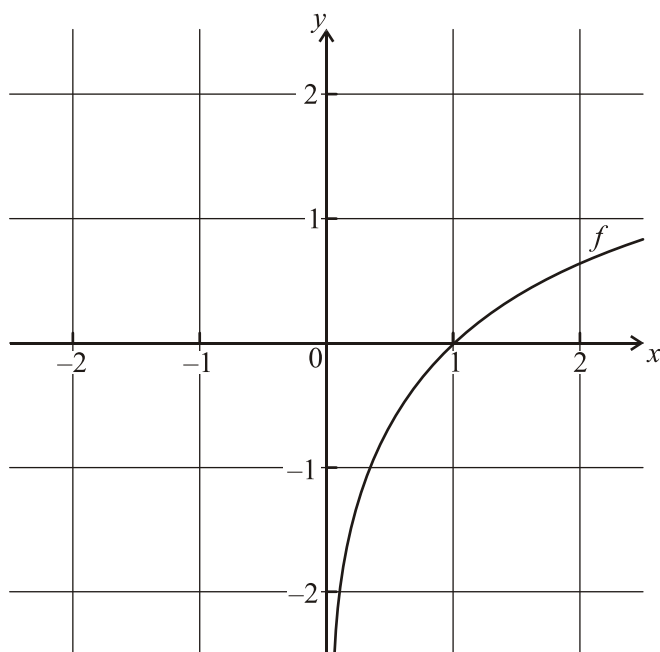
(i) $f(a)$;

(ii) $f(1)$;

(iii) $f(a^4)$.

(3)

(b) The diagram below shows part of the graph of f .



On the same diagram, sketch the graph of f^{-1} .

(3)
(Total 6 marks)

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6. Given that $\log_5 x = y$, express each of the following in terms of y .

(a) $\log_5 x^2$

(b) $\log_5 \left(\frac{1}{x} \right)$

(c) $\log_{25} x$

Working:

Answers:

(a)

(b)

(c)

(Total 6 marks)

7. (a) Given that $\log_3 x - \log_3 (x - 5) = \log_3 A$, express A in terms of x .

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3 (x - 5) = 1$.

Working:

Answers:

(a)

(b)

(Total 6 marks)

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8. (a) Find $\log_2 32$.

(1)

(b) Given that $\log_2 \left(\frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of p and of q .

(4)

(Total 5 marks)

9. Let $\log_{10}P = x$, $\log_{10}Q = y$ and $\log_{10}R = z$. Express $\log_{10} \left(\frac{P}{QR^3} \right)^2$ in terms of x , y and z .

Working:

Answer:

.....

(Total 4 marks)

10. Solve the following equations.

(a) $\ln(x + 2) = 3$.

(b) $10^{2x} = 500$.

(Total 6 marks)

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11. Let $a = \log x$, $b = \log y$, and $c = \log z$.

Write $\log \left(\frac{x^2 \sqrt{y}}{z^3} \right)$ in terms of a , b and c .

Working:

Answer:

.....
(Total 6 marks)

12. Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

Working:

Answer:

.....
(Total 6 marks)

14. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$;

(ii) $\log_c 25$.

- (b) Find the value of d if $\log_d 6 = \frac{1}{2}$.

(Total 6 marks)

18. Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q .

(a) $\ln a^3 b$

(b) $\ln \left(\frac{\sqrt{a}}{b} \right)$

(Total 6 marks)