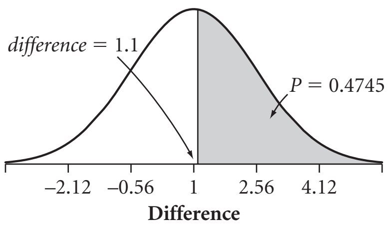
E60. The cap is too loose if $d_c - d_b > 1.1$. From E59, the sampling distribution of the difference is approximately normal and has a mean of 1 and a standard error of 1.562. A difference of 1.1 has a z-score of

$$z = \frac{1.1 - 1}{1.562} \approx 0.064$$



The probability that the cap is too loose is about 0.4745.

E61. **a.** $\mu_{\text{morning}} = 12.8$ and $\sigma^2_{\text{morning}} = 4.56$; $\mu_{\text{afternoon}} = 8$ and $\sigma^2_{\text{afternoon}} = 4$

b. $\mu_{\text{total}} = 20.8$ and $\sigma^2_{\text{total}} = 6.96$

c. The sum of the means 12.8 + 8 equals 20.8, as we would expect. But the sum of the variances 4.56 + 4 is not equal to 6.96, and we do not expect it to be because the morning and afternoon times were not selected independently when we computed 6.96.

AP Sample Test

AP1. C. There are three possible samples with a median of 1, six samples with a median of 1.5, and one sample with a median of 2, so the median of the sampling distribution is the median of those ten values, which is 1.5.

AP2. B. The sample maximum is always less than or equal to the population maximum, never greater, so it is a biased estimator.

AP3. D. See E11 on pages 424–425.

AP4. D. The mean of the sampling distribution is always equal to the mean of the population. The standard deviation of the sampling distribution, often called the standard error, is the population standard deviation divided by the square root of n, so it gets smaller as the sample size increases. The Central Limit Theorem guarantees that the shape will get closer to normal.

AP5. B. The mean of the sampling distribution is 500 with a standard deviation of $\frac{110}{\sqrt{n}} = \frac{110}{\sqrt{100}} = 11$. The probability that the mean is larger than 510 is then the probability that z is larger than $(510 - 500)/11$, which is approximately 0.1817.

AP6. D. The expected number correct is 10, and the standard deviation is $\sqrt{40 \cdot 0.25 \cdot 0.75} \approx 2.74$, so $z = (15 - 10)/2.74 \approx 1.83$.

AP7. B. Here $z = \frac{0.24 - 0.20}{\sqrt{\frac{(0.2)(0.8)}{1200}}} \approx 3.46$, and because the distribution of the sample proportion is approximately normal, this leads to a probability of about 0.0003.

AP8. D. If the sample size is large enough or if the population is close enough to normal, then there will be an approximately normal shape for the sampling distribution of the mean (or the total: they're the same shape, because the only difference is a multiplication by n). However, here n is only 2, and the population of sleep times might not be very close to normal—in fact it is probably left-skewed, because the mean is so close to the maximum of 360 minutes. Note that students may argue that "randomly selected" doesn't imply independent selection (although it usually does) and so *C* would be a correct answer. [*Source:* pediatrics.aappublications.org]

AP9. **a.** The best estimate of the number going straight on an average day—under the assumption that each car is equally likely to go each direction—is

$$246{,}000\left(\frac{1}{3}\right) = 82{,}000$$

b. The sampling distribution of the sample total should be approximately normal with a mean of 82,000 and a standard error

$$\sqrt{246{,}000\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} \approx 233.809$$

The range of reasonably likely outcomes includes values within 1.96 standard errors of the mean:

$$82{,}000 \pm 1.96(233.809)$$

In other words, the probability is 0.95 that between 81,542 and 82,548 vehicles will go straight through on an average day if it is true that a randomly selected vehicle has a $\frac{1}{3}$ chance of going straight through the interchange.

c. Having an average of 138,300 vehicles go straight through the interchange is not a reasonably likely outcome under the assumption that vehicles are equally likely to go in each of the three directions. This assumption cannot be even approximately true.

AP10. **a.** This problem can be solved using either the sampling distribution of the mean weight or the sampling distribution of the total weight. We will use the mean weight. The sampling distribution for the mean weight of n people has a mean of 150 and standard error $\sigma_{\bar{x}} = 20/\sqrt{n}$.

If you choose $n = 14$: The mean weight must be less than $\frac{2000}{14} \approx 142.86$. This is 1.34 standard errors below the expected mean weight of 150. Thus, it will be exceeded about 91% of the time. The elevator is almost sure to be overloaded.

If you choose $n = 13$: The mean weight must be less than $\frac{2000}{13} \approx 153.85$. This is 0.694 standard error above the expected mean weight of 150. Thus, the elevator will be overloaded about 24% of the time.

If you choose $n = 12$: The mean weight must be less than $\frac{2000}{12} \approx 166.67$. This is 2.89 standard errors above the expected mean weight of 150, and the elevator will be overloaded about 0.19% of the time.

It appears that $n = 12$ would be a good choice for the maximum occupancy if the consequences could be severe with an occasionally overloaded elevator.

This reasoning assumes that the sample of people who get on the elevator are selected randomly from the population. If you expect for some reason that the sample of people who get on the elevator may not be random, then these results will not apply. For example, if a weight loss clinic for men is on the upper floor and large groups of men tend to leave together after an exercise class, the "sample" of people would not be random and the elevator would be overloaded every time they left.

b. These are almost the same numbers that we found in part a. It's somewhat surprising that Otis's conclusion is this close to ours as their estimates probably were made using different data and different assumptions. For example, Mitsubishi uses an average weight of 65 kg (about 143 pounds) per person for its elevators.