Series and Sequences, Binomial Theorem Review Paper 2 KEY

1. $(3x + 2y)^4 = (3x)^4 + \binom{4}{1}(3x)^2(2y) + \binom{4}{2}(3x)^2(2y)^2 + \binom{4}{3}(3x)(2y)^3 + (2y)^4$ (A1)

$= 81x^4 + 216x^3y + \mathbf{216x^2y^2 + 96xy^3 + 16y^4}$ (A1)(A1)(A1) (C4)

[4]

2. The constant term will be the term independent of the variable x. (R1)

$\left(x - \frac{2}{x^2}\right)^9 = x^9 + 9x^8\left(\frac{-2}{x^2}\right) + \ldots + \binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3 + \ldots + \left(\frac{-2}{x^2}\right)^9$ (M1)

$\binom{9}{3}x^6\left(\frac{-2}{x^2}\right)^3 = 84x^6\left(\frac{-8}{x^6}\right)$ (A1)

$= -672$ (A1)

[4]

3. Selecting one term (may be implied) (M1)

$\binom{7}{2}5^2(2x^2)^5$ (A1)(A1)(A1)

$= 16800x^{10}$ (A1)(A1) (C6)

Note: *Award C5 for 16800.*

[6]

4. Required term is $\binom{8}{5}(3x)^5(-2)^3$ (A1)(A1)(A1)

Therefore the coefficient of x^5 is $56 \times 243 \times -8$

$= -108864$ (A1) (C4)

[4]

5. $(5a + b)^7 = \ldots + \binom{7}{4}(5a)^3(b)^4 + \ldots$ (M1)

$= \frac{7 \times 6 \times 5 \times 4}{1 \times 2 \times 3 \times 4} \times 5^3 \times (a^3b^4) = 35 \times 5^3 \times a^3b^4$ (M1)(A1)

So the coefficient is 4375 (A1) (C4)

[4]

6. $(a + b)^{12}$

Coefficient of a^5b^7 is $\binom{12}{5} = \binom{12}{7}$ (M1)(A1)

$= 792$ (A2) (C4)

[4]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

7. (a) 10 (A2) (C2)

(b) $(3x^2)^3\left(-\frac{1}{x}\right)^6$ [for correct exponents] (M1)(A1)

$\binom{9}{6}3^3x^6\frac{1}{x^6}\left(\text{or } 84\times3^3x^6\frac{1}{x^6}\right)$ (A1)

constant = 2268 (A1) (C4)

[6]

8. (a) For taking three ratios of consecutive terms (M1)

$\frac{54}{18}=\frac{162}{54}=\frac{486}{162}\ (=3)$ A1

hence geometric AG N0

(b) (i) $r = 3$ (A1)

$u_n = 18 \times 3^{n-1}$ A1 N2

(ii) For a valid attempt to solve $18 \times 3^{n-1} = 1062882$ (M1)

eg trial and error, logs

$n = 11$ A1 N2

[6]

9. (a) $\$1000 \times 1.075^{10} = \$2061$ (nearest dollar) (A1) (C1)

(b) $1000(1.075^{10} + 1.075^9 + ... + 1.075)$ (M1)

$= \frac{1000(1.075)(1.075^{10}-1)}{1.075-1}$ (M1)

= \$15208 (nearest dollar) (A1) (C3)

[4]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

10. (a) $5000(1.063)^n$ A1 1

(b) Value = $\$5000(1.063)^5$ (= \$6786.3511...)
= \$6790 to 3 sf (Accept \$6786, or \$6786.35) A1 1

(c) (i) $5000(1.063)^n > 10000$ or $(1.063)^n > 2$ A1 1

(ii) Attempting to solve the inequality «log (1.063) > log 2 (M1)
$n > 11.345...$ (A1)
12 years A1 3

__Note__: Candidates are likely to use TABLE or LIST on a GDC to find n. A good way of communicating this is suggested below.

Let $y = 1.063^x$ (M1)
When $x = 11$, $y = 1.9582$, when $x = 12$, $y = 2.0816$ (A1)
$x = 12$ *ie* 12 years A1 3

[6]

11. (a) evidence of substituting into formula for *n*th term of GP (M1)

e.g. $u_4 = \frac{1}{81}r^3$

setting up correct equation $\frac{1}{81}r^3 = \frac{1}{3}$ A1

$r = 3$ A1 N2

(b) **METHOD 1**

setting up an inequality (accept an equation) M1

e.g. $\frac{\frac{1}{81}(3^n - 1)}{2} > 40; \frac{\frac{1}{81}(1-3^n)}{-2} > 40; 3^n > 6481$

evidence of solving M1
e.g. graph, taking logs

$n > 7.9888...$ (A1)
$n = 8$ A1 N2

METHOD 2

if $n = 7$, sum = 13.49...; if $n = 8$, sum = 40.49... A2

$n = 8$ (is the smallest value) A2 N2

[7]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

12. (a) common difference is 6 — A1 N1

(b) evidence of appropriate approach — (M1)
e.g. $u_n = 1353$

correct working — A1

e.g. $1353 = 3 + (n - 1)6$, $\frac{1353+3}{6}$

$n = 226$ — A1 N2

(c) evidence of correct substitution — A1

e.g. $S_{226} = \frac{226(3+1353)}{2}, \frac{226}{2}(2 \times 3 + 225 \times 6)$

$S_{226} = 153\ 228$ (accept 153 000) — A1 N1

[6]

13. (a) (i) $r = -2$ — A1 N1

(ii) $u_{15} = -3\,(-2)^{14}$ — (A1)

$= -49152$ (accept -49200) — A1 N2

(b) (i) 2, 6, 18 — A1 N1

(ii) $r = 3$ — A1 N1

(c) Setting up equation (or a sketch) — M1

$\frac{x+1}{x-3} = \frac{2x+8}{x+1}$ (or correct sketch with relevant information) — A1

$x^2 + 2x + 1 = 2x^2 + 2x - 24$ — (A1)

$x^2 = 25$

$x = 5$ or $x = -5$

$x = -5$ — A1 N2

***Notes**: If "trial and error" is used, work must be documented with several trials shown.*
Award full marks for a correct answer with this approach.
*If the work is **not** documented, award N2 for a correct answer.*

(d) (i) $r = \frac{1}{2}$ — A1 N1

(ii) For attempting to use infinite sum formula for a GP — (M1)

$S = \frac{-8}{1-\frac{1}{2}}$

$S = -16$ — A1 N2

***Note:** Award M0A0 if candidates use a value of r where r > 1, or r < –1.*

[12]

14. (a) evidence of equation for u_{27} M1

e.g. $263 = u_1 + 26 \times 11$, $u_{27} = u_1 + (n - 1) \times 11$, $263 - (11 \times 26)$

$u_1 = -23$ A1 N1

(b) (i) correct equation A1

e.g. $516 = -23 + (n - 1) \times 11$, $539 = (n - 1) \times 11$

$n = 50$ A1 N1

(ii) correct substitution into sum formula A1

e.g. $S_{50} = \dfrac{50(-23+516)}{2}$, $S_{50} = \dfrac{50(2\times(-23)+49\times 11)}{2}$

$S_{50} = 12325$ (accept 12300) A1 N1

[6]

15. **METHOD 1**

substituting into formula for S_{40} (M1)

correct substitution A1

e.g. $1900 = \dfrac{40(u_1 + 106)}{2}$

$u_1 = -11$ A1 N2

substituting into formula for u_{40} or S_{40} (M1)

correct substitution A1

e.g. $106 = -11 + 39d$, $1900 = 20(-22 + 39d)$

$d = 3$ A1 N2

METHOD 2

substituting into formula for S_{40} (M1)

correct substitution A1

e.g. $20(2u_1 + 39d) = 1900$

substituting into formula for u_{40} (M1)

correct substitution A1

e.g. $106 = u_1 + 39d$

$u_1 = -11$, $d = 3$ A1A1 N2N2

[6]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

16. (a) $r = \frac{360}{240} = \frac{240}{160} = \frac{3}{2} = 1.5$ (A1) 1

(b) 2002 is the 13th year. (M1)

$u_{13} = 160(1.5)^{13-1}$ (M1)

= 20759 (Accept 20760 or 20800.) (A1) 3

(c) $5000 = 160(1.5)^{n-1}$

$\frac{5000}{160} = (1.5)^{n-1}$ (M1)

$\log\left(\frac{5000}{160}\right) = (n-1)\log 1.5$ (M1)

$n - 1 = \frac{\log\left(\frac{5000}{160}\right)}{\log 1.5} = 8.49$ (A1)

$\Rightarrow n = 9.49 \Rightarrow 10^{\text{th}}$ year

$\Rightarrow 1999$ (A1)

OR

Using a gdc with $u_1 = 160$, $u_{k+1} = \frac{3}{2}u_k$, $u_9 = 4100$, $u_{10} = 6150$ (M2)

1999 (G2) 4

(d) $S_{13} = 160\left[\frac{1.5^{13} - 1}{1.5 - 1}\right]$ (M1)

= 61958 (Accept 61960 or 62000.) (A1) 2

(e) Nearly everyone would have bought a portable telephone so there would be fewer people left wanting to buy one. (R1)

OR

Sales would saturate. (R1) 1

[11]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

17. (a) (i) $PQ = \sqrt{AP^2 + AQ^2}$ (M1)

$= \sqrt{2^2 + 2^2} = \sqrt{4(2)} = 2\sqrt{2}$ cm (A1)(AG)

(ii) Area of PQRS = $(2\sqrt{2})(2\sqrt{2}) = 8$ cm^2 (A1) 3

(b) (i) Side of third square = $\sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2$ cm

Area of third square = 4 cm^2 (A1)

(ii) $\frac{1^{st}}{2^{nd}} = \frac{16}{8}$ $\frac{2^{nd}}{3^{rd}} = \frac{8}{4}$ (M1)

$\Rightarrow$ Geometric progression, $r = \frac{8}{16} = \frac{4}{8} = \frac{1}{2}$ (A1) 3

(c) (i) $u_{11} = u_1 r^{10} = 16\left(\frac{1}{2}\right)^{10} = \frac{16}{1024}$ (M1)

$= \frac{1}{64}$ (= 0.015625 = 0.0156, 3 sf) (A1)

(ii) $S_\infty = \frac{u_1}{1-r} = \frac{16}{1-\frac{1}{2}}$ (M1)

$= 32$ (A1) 4

[10]

18. (a) Ashley

AP 12 + 14 + 16 + ... to 15 terms (M1)

$S_{15} = \frac{15}{2}[2(12) + 14(2)]$ (M1)

$= 15 \times 26$

$= 390$ hours (A1) 3

(b) Billie

GP 12, 12(1.1), $12(1.1)^2$… (M1)

(i) In week 3, $12(1.1)^2$ (A1)

$= 14.52$ hours (AG)

(ii) $S_{15} = \frac{12[(1.1)^{15} - 1]}{1.1 - 1}$ (M1)

$= 381$ hours (3 sf) (A1) 4

(c) $12\,(1.1)^{n-1} > 50$ (M1)

$(1.1)^{n-1} > \frac{50}{12}$ (A1)

$(n - 1)\ln 1.1 > \ln\frac{50}{12}$

$n - 1 > \frac{\ln\frac{50}{12}}{\ln 1.1}$ (A1)

$n - 1 > 14.97$

$n > 15.97$

$\Rightarrow$ Week 16 (A1)

OR

$12(1.1)^{n-1} > 50$ (M1)

By trial and error

$12(1.1)^{14} = 45.6$, $12(1.1)^{15} = 50.1$ (A1)

$\Rightarrow n - 1 = 15$ (A1)

$\Rightarrow n = 16$ (Week 16) (A1) 4

[11]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

19. (a) (i) Area B = $\frac{1}{16}$, area C = $\frac{1}{64}$ (A1)(A1)

(ii) $\frac{\frac{1}{16}}{\frac{1}{4}} = \frac{1}{4}$ $\frac{\frac{1}{64}}{\frac{1}{16}} = \frac{1}{4}$ (Ratio is the same.) (M1)(R1)

(iii) Common ratio = $\frac{1}{4}$ (A1) 5

(b) (i) Total area (S_2) = $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$ = (= 0.3125) (0.313, 3 sf) (A1)

(ii) Required area = S_8 = $\dfrac{\frac{1}{4}\left(1-\left(\frac{1}{4}\right)^8\right)}{1-\frac{1}{4}}$ (M1)

= 0.333328 2(471...) (A1)

= 0.333328 (6 sf) (A1) 4

Note: *Accept result of adding together eight areas correctly.*

(c) Sum to infinity = $\dfrac{\frac{1}{4}}{1-\frac{1}{4}}$ (A1)

= $\frac{1}{3}$ (A1) 2

[11]

Series and Sequences, Binomial Theorem Review Paper 2 KEY

20. (a) (i) \$11400, \$11800 (A1) 1

(ii) Total salary $= \frac{10}{2}(2 \times 11\,000 + 9 \times 400)$ (A1)

$= \$128000$ (A1) (N2) 2

(b) (i) \$10700, \$11449 (A1)(A1)

(ii) 10^{th} year salary $= 10\,000(1.07)^9$ (A1)

$= \$18384.59$ or $\$18400$ or $\$18385$ (A1) (N2) 4

(c) **EITHER**

Scheme A $S_A = \frac{n}{2}\left(2 \times 11\,000 + (n-1)400\right)$ (A1)

Scheme B $S_B = \frac{10\,000(1.07^n - 1)}{1.07 - 1}$ (A1)

Solving $S_B > S_A$ (accept $S_B = S_A$, giving $n = 6.33$) (may be implied) (M1)

Minimum value of n is 7 years. (A1) (N2)

OR

Using trial and error (M1)

	Arturo	Bill
6 years	\$72 000	\$71532.91
7 years	\$85 400	\$86 540.21

(A1)(A1)

Note: *Award (A1) for **both** values for 6 years, and (A1) for **both** values for 7 years.*

Therefore, minimum number of years is 7. (A1) (N2) 4

[11]