Series and Sequences, Binomial Theorem Review Paper 1 KEY

1. (a) $u_{10} = 3(0.9)^9$ A1 N1

(b) recognizing $r = 0.9$ (A1)
correct substitution A1
e.g. $S = \frac{3}{1-0.9}$
$S = \frac{3}{0.1}$ (A1)
$S = 30$ A1 N3

[5]

2. (a) attempt to find d (M1)
e.g. $\frac{u_3 - u_1}{2}$, $8 = 2 + 2d$
$d = 3$ A1 N2 2

(b) correct substitution (A1)
e.g. $u_{20} = 2 + (20 - 1)3$, $u_{20} = 3 \times 20 - 1$
$u_{20} = 59$ A1 N2 2

(c) correct substitution (A1)
e.g. $S_{20} = \frac{20}{2}(2 + 59)$, $S_{20} = \frac{20}{2}(2 \times 2 + 19 \times 3)$
$S_{20} = 610$ A1 N2 2

[6]

3. (a) $u_1 = 1$, $u_2 = -1$, $u_3 = -3$ A1A1A1 N3

(b) Evidence of using appropriate formula M1
correct values $S_{20} = \frac{20}{2}(2 \times 1 + 19 \times -2)$ $(= 10(2 - 38))$ A1
$S_{20} = -360$ A1 N1

[6]

4. (a) $\frac{1}{5}$ (0.2) A1 N1

(b) (i) $u_{10} = 25\left(\frac{1}{5}\right)^9$ (M1)

$= 0.0000128 \left(\left(\frac{1}{5}\right)^7, 1.28\times10^{-5}, \frac{1}{78125}\right)$ A1 N2

(ii) $u_n = 25\left(\frac{1}{5}\right)^{n-1}$ A1 N1

(c) For attempting to use infinite sum formula for a GP $\left(\frac{25}{1-\left(\frac{1}{5}\right)}\right)$ (M1)

$S = \frac{125}{4} = 31.25 \ (= 31.3 \text{ to } 3\,s\,f)$ A1 N2

[6]

5. (a) 3, 6, 9 A1 N1

(b) (i) Evidence of using the sum of an AP M1

eg $\frac{20}{2}\ 2\times3 + (20-1)\times3$

$\sum_{n=1}^{20} 3n = 630$ A1 N1

(ii) **METHOD 1**

Correct calculation for $\sum_{n=1}^{100} 3n$ (A1)

eg $\frac{100}{2}(2\times3 + 99\times3), 15150$

Evidence of subtraction (M1)

eg 15150 – 630

$\sum_{n=21}^{100} 3n = 14520$ A1 N2

METHOD 2

Recognising that first term is 63, the number of terms is 80 (A1)(A1)

eg $\frac{80}{2}(63+300), \frac{80}{2}(126+79\times3)$

$\sum_{n=21}^{100} 3n = 14520$ A1 N2

[6]

6. (a) For taking an appropriate ratio of consecutive terms (M1)

$r = \frac{2}{3}$ A1 N2

(b) For attempting to use the formula for the n^{th} term of a GP (M1)

$u_{15} = 1.39$ A1 N2

(c) For attempting to use infinite sum formula for a GP (M1)

$S = 1215$ A1 N2

[6]

7. For using $u_3 = u_1 r^2 = 8$ (M1)

$8 = 18r^2$ (A1)

$r^2 = \frac{8}{18}\left(=\frac{4}{9}\right)$

$r = \pm\frac{2}{3}$ (A1)(A1)

$S_\infty = \frac{u_1}{1-r}$,

$S_\infty = 54, \frac{54}{5}(=10.8)$ (A1)(A1)(C3)(C3)

[6]

8. Arithmetic sequence $d = 3$ (may be implied) (M1)(A1)

$n = 1250$ (A2)

$S = \frac{1250}{2}(3 + 3750)$ $\left(\text{or } S = \frac{1250}{2}(6 + 1249 \times 3)\right)$ (M1)

$= 2\ 345\ 625$ (A1) (C6)

[6]

9. Arithmetic sequence (M1)

$a = 200$ $d = 30$ (A1)

(a) Distance in final week = $200 + 51 \times 30$ (M1)

$= 1730$ m (A1) (C3)

(b) Total distance = $\frac{52}{2}[2.200 + 51.30]$ (M1)

$= 50180$ m (A1) (C3)

Note: *Penalize once for absence of units ie award A0 the first time units are omitted, A1 the next time.*

[6]

10. (a) $u_4 = u_1 + 3d$ or $16 = -2 + 3d$ (M1)

$d = \dfrac{16-(-2)}{3}$ (M1)

$= 6$ (A1) (C3)

(b) $u_n = u_1 + (n-1)6$ or $11998 = -2 + (n-1)6$ (M1)

$n = \dfrac{11998+2}{6} + 1$ (A1)

$= 2001$ (A1) (C3)

[6]

11. $S = \dfrac{u_1}{1-r} = \dfrac{\frac{2}{3}}{1-\left(-\frac{2}{3}\right)}$ (M1)(A1)

$= \dfrac{2}{3} \times \dfrac{3}{5}$ (A1)

$= \dfrac{2}{5}$ (A1) (C4)

[4]

12. $S_5 = \dfrac{5}{2}\{2 + 32\}$ (M1)(A1)(A1)

$S_5 = 85$ (A1)

OR

$a = 2,\ a + 4d = 32$ (M1)

$\Rightarrow 4d = 30$

$d = 7.5$ (A1)

$S_5 = \dfrac{5}{2}(4 + 4(7.5))$ (M1)

$= \dfrac{5}{2}(4 + 30)$

$S_5 = 85$ (A1) (C4)

[4]