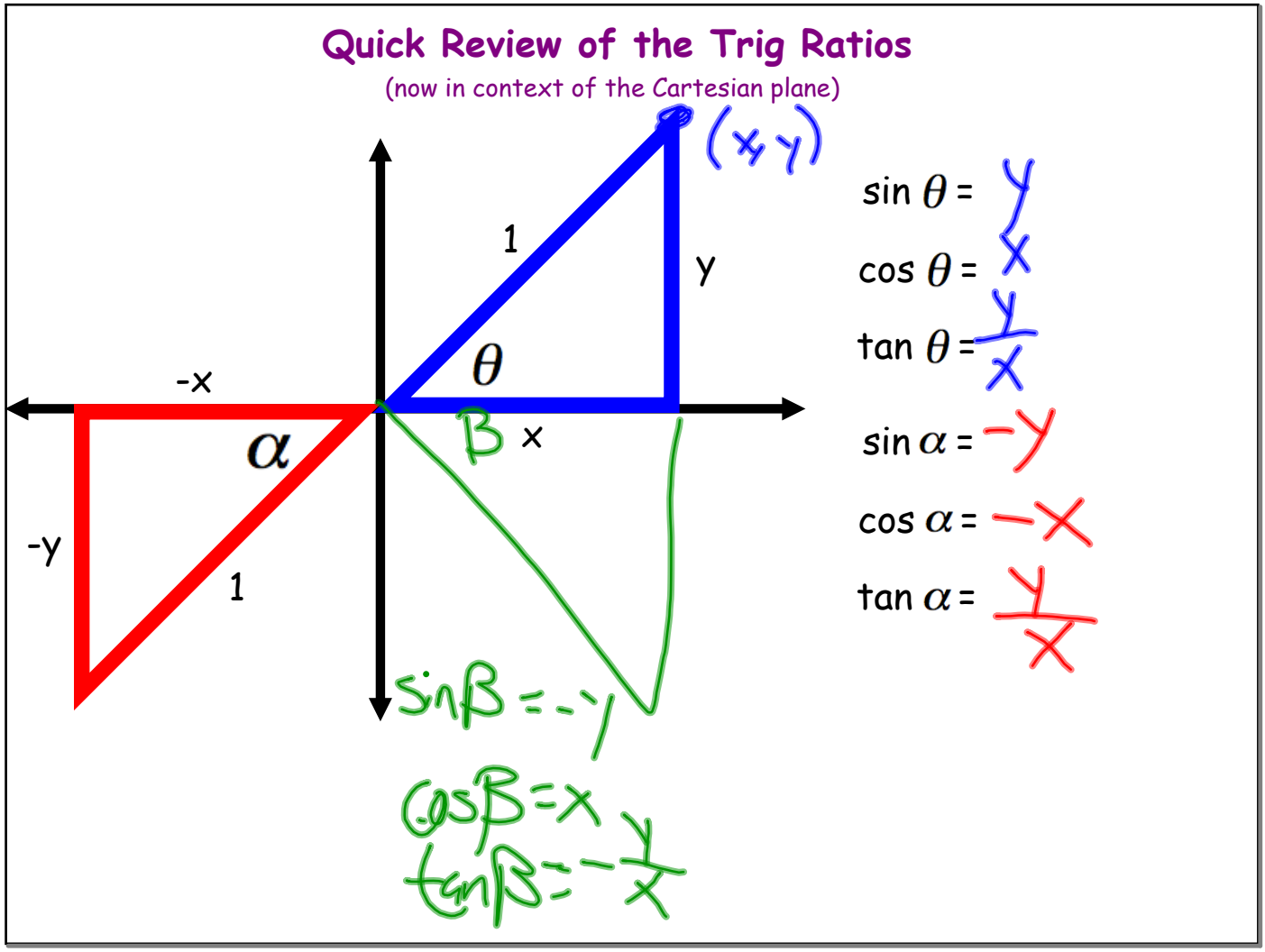
The Unit Circle

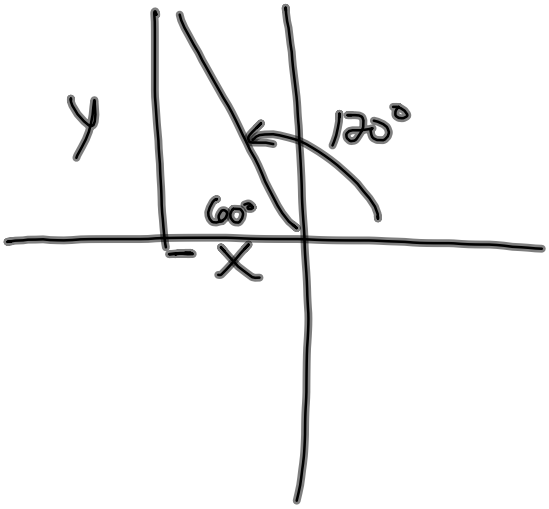
The Unit Circle is created by multiples of $30^\circ, 45^\circ, 60^\circ$, and 90° or $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ angles

We only need these values to find ALL of the values on the Unit Circle.


Quick Review of the Trig Ratios
(now in context of the Cartesian plane)
(x, y)
1
y
θ
-x
α
B
x
-y
1
sin θ = y
cos θ = x
tan θ = y/x
sin α = -y
cos α = -x
tan α = y/x
sinB = -y
cosB = x
tanB = -y/x

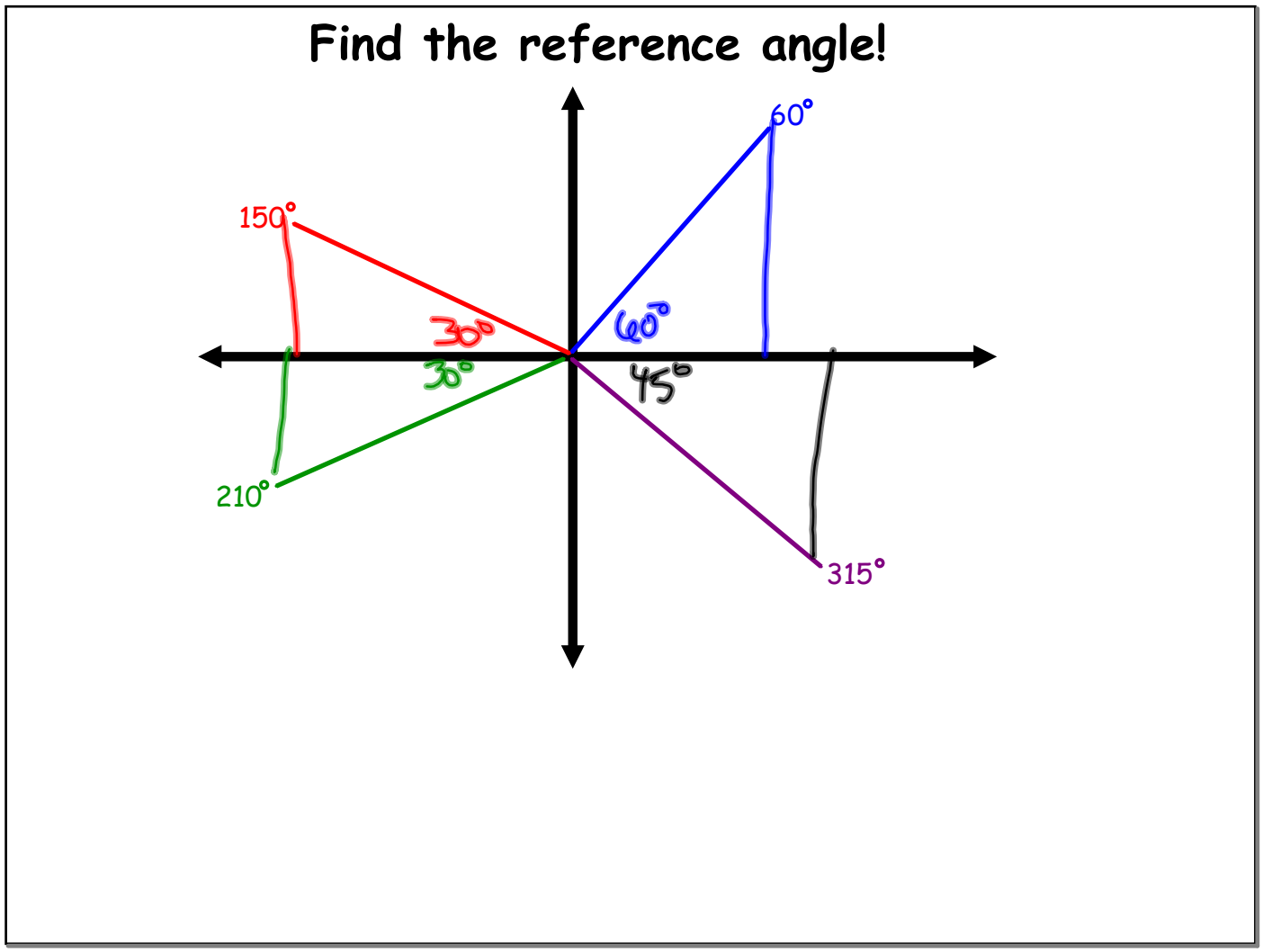
Why do we only need some angles?

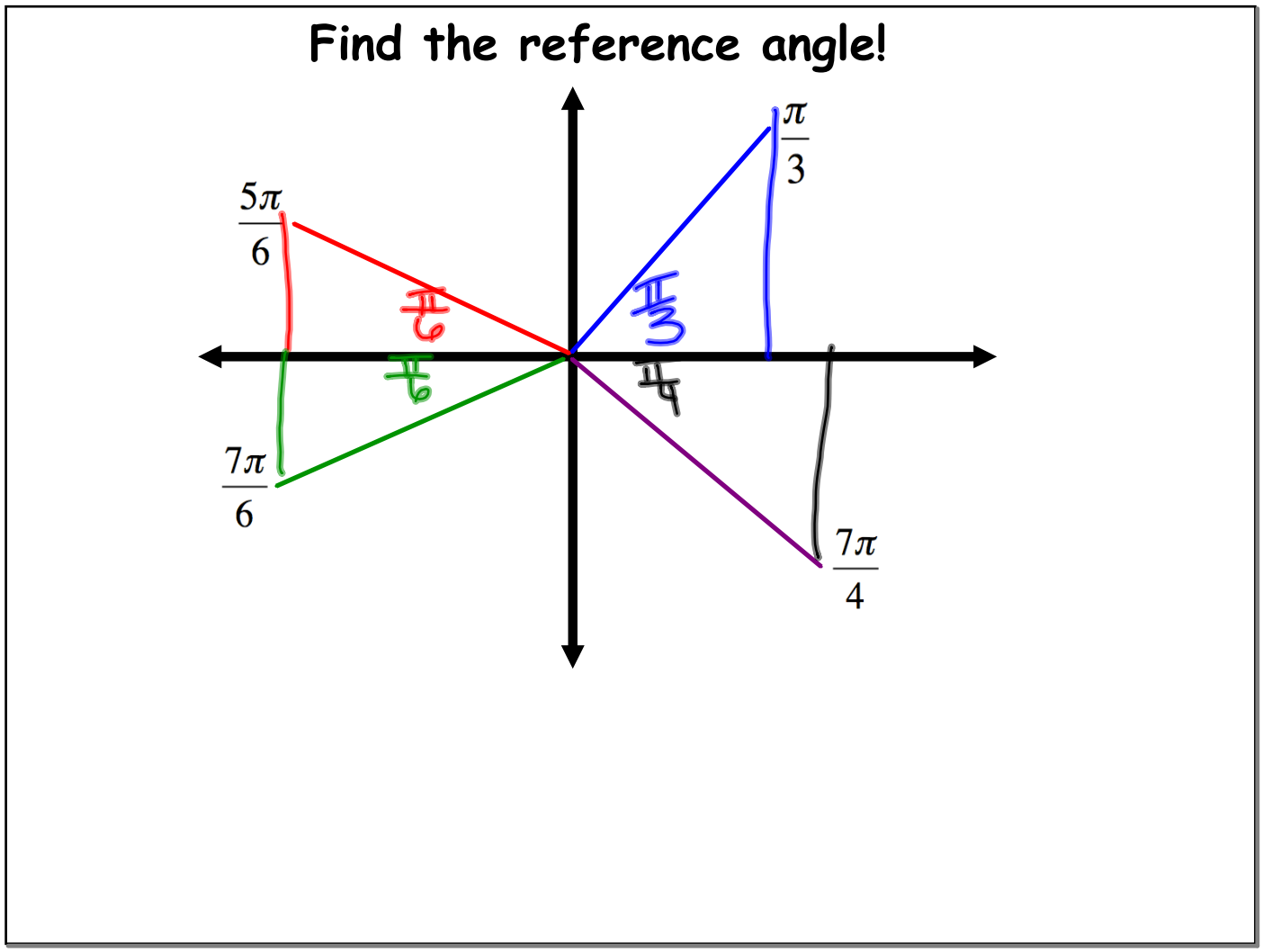
Sketch: 120°



Can we set up a right triangle with an angle of 120°?

Where could we draw an angle that CAN be used in a right triangle but is related to the original angle?

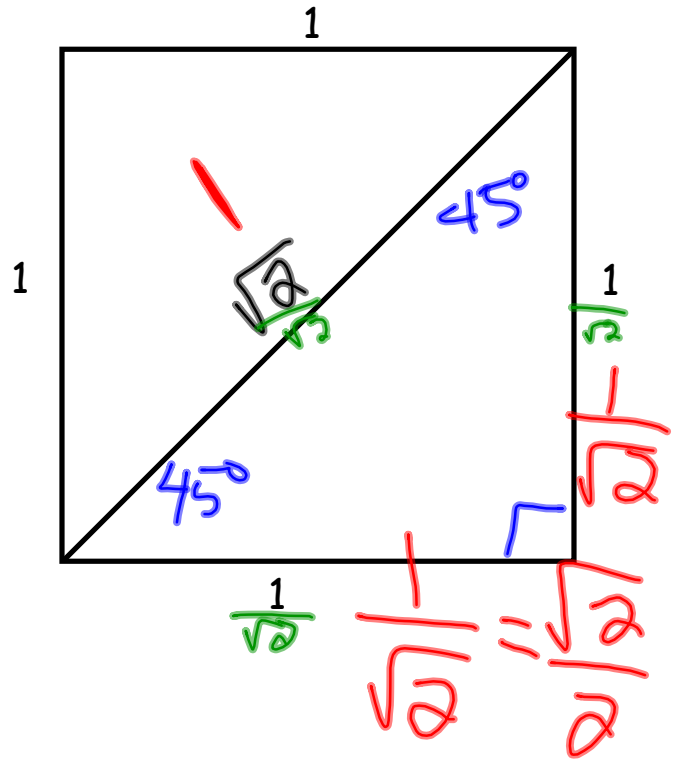

Find the reference angle!
150°
30°
30°
210°
60°
60°
45°
315°


Find the reference angle!
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
$\frac{\pi}{6}$
$\frac{7\pi}{6}$
$\frac{\pi}{3}$
$\frac{\pi}{3}$
$\frac{\pi}{4}$
$\frac{7\pi}{4}$

Find the reference angles for
the angles on your homework!

Remember: 45°-45°-90° Triangle

$\frac{\pi}{4}$ $\frac{\pi}{4}$ $\frac{\pi}{2}$



$\sin(45°) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

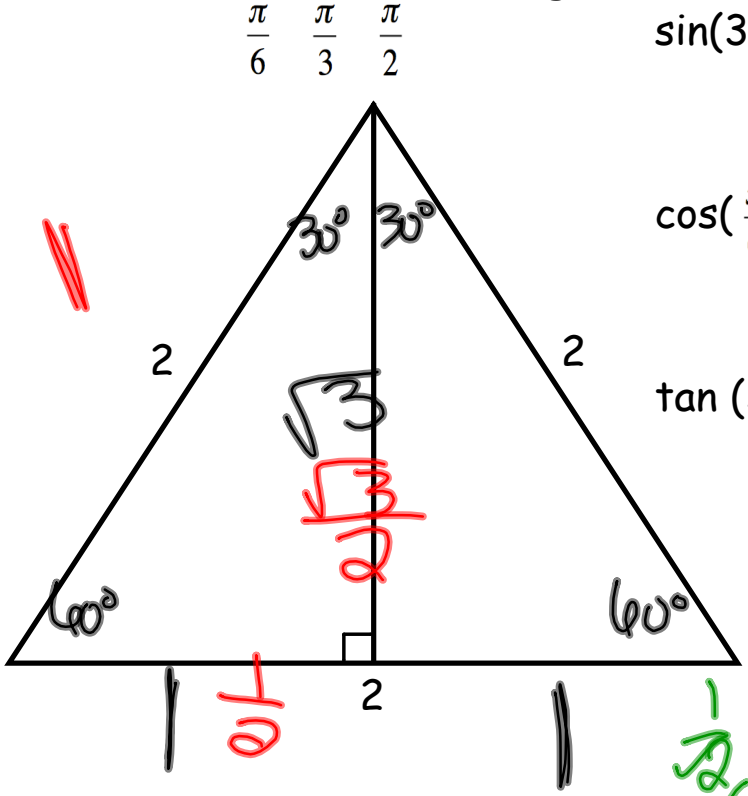
$\cos(45°) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\tan(45°) = 1$

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Remember: 30°-60°-90° Triangle

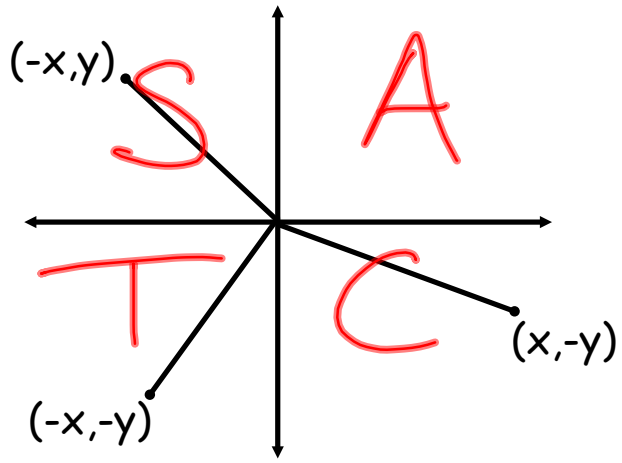


$\sin(30°) = \frac{1}{2}$ $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

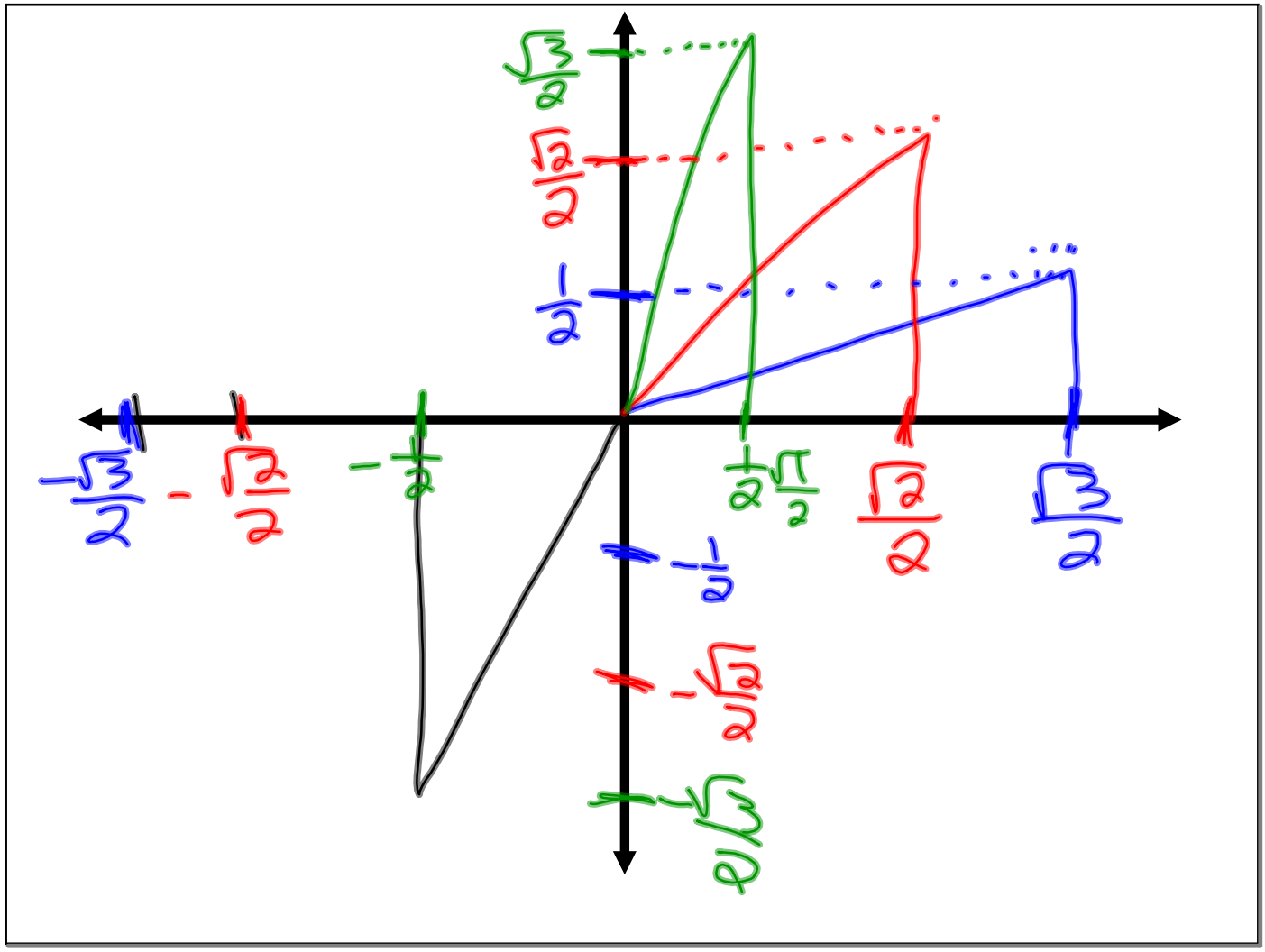
$\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ $\cos(60°) = \frac{1}{2}$

$\tan(30°) = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\tan(\frac{\pi}{3}) = \sqrt{3}$

What happens when we move out of the first quadrant?



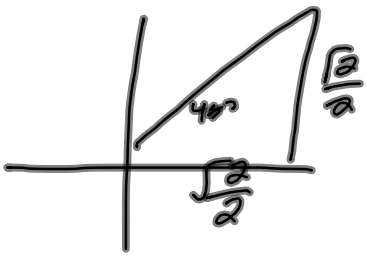
Ratios are the SAME, the signs change depending on the quadrant!


$\frac{\sqrt{3}}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{1}{2}$
$-\frac{\sqrt{3}}{2}$
$-\frac{\sqrt{2}}{2}$
$-\frac{1}{2}$
$\frac{1}{2}$
$\frac{\sqrt{2}}{2}$
$\frac{\sqrt{3}}{2}$
$-\frac{1}{2}$
$-\frac{\sqrt{2}}{2}$
$-\frac{\sqrt{3}}{2}$

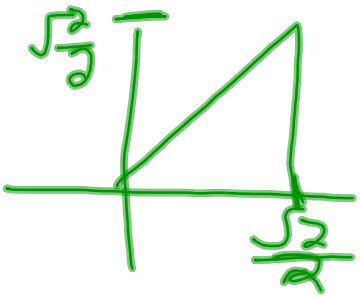
So how do we do this with ANY angle?

1. Draw a sketch of where the angle is in the coordinate plane.
2. Create a triangle to the x-axis.
3. Determine the reference angle, and label the sides of the triangle.
4. Find the value of the trig ratio!

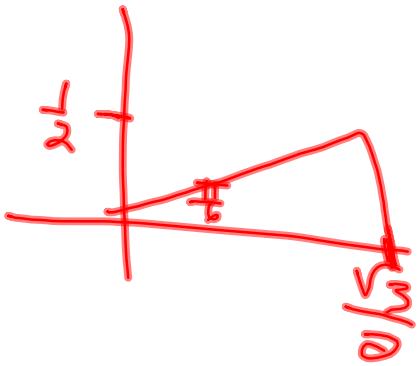
$\sin(45°) = \frac{\sqrt{2}}{2}$



$\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$



$\tan(\frac{\pi}{3}) = \sqrt{3}$



$\sin(135^\circ) = \frac{\sqrt{2}}{2}$

$\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$

$\sin(300^\circ) = -\frac{\sqrt{3}}{2}$

$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2}$

$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$\cos(150^\circ) = -\frac{\sqrt{3}}{2}$

$\tan(120^\circ) = -\sqrt{3}$

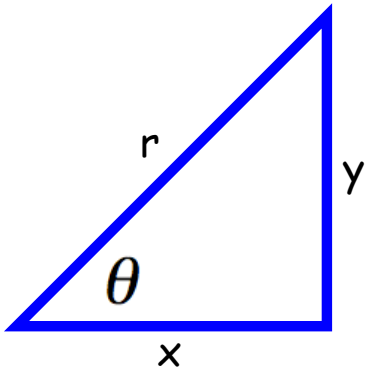
$\sin\left(\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\tan(240^\circ) = \sqrt{3}$

What about the angles on the axis?

These are called your quadrantal angles

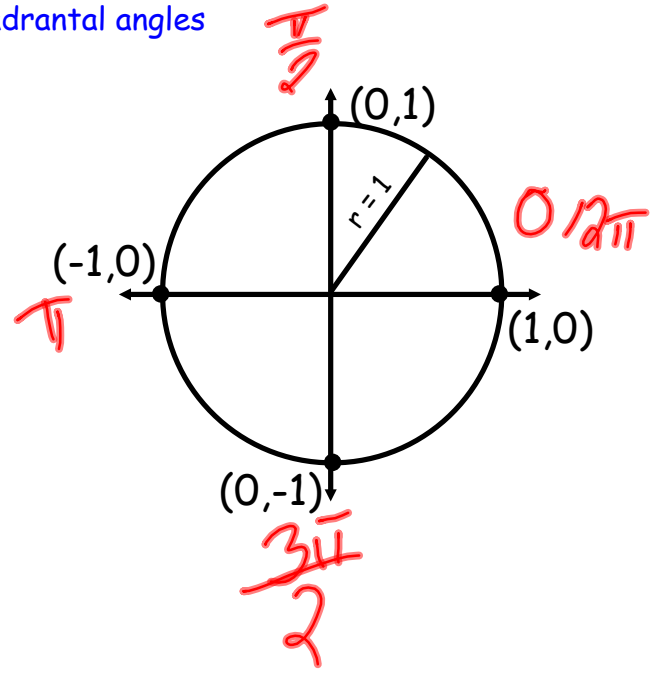
Remember our trig ratios in context of the Cartesian plane:



$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{x}$



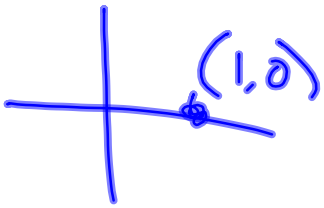
So what are the following:

$\sin(270^\circ) =$ -1

$\sin\left(\frac{\pi}{2}\right) =$ 1

$\cos(\pi) =$ -1

$\cos(360^\circ) =$ 1



$\tan\left(\frac{3\pi}{2}\right) =$ Undef

Homework

Unit Circle Worksheet

(practice method, don't copy from notes!)