Applications of Exponential and Logarithmic Functions

Growth & Decay

Here are just a few applications of exponential growth and decay models.

Biology
- Growth of micro-organisms in a culture
- Human population
- Spread of a virus

Physics
- Nuclear chain reactions
- Heat transfer

Economics
- Pyramid schemes

Computer technology
- Processing power of computers
- Internet traffic growth

You may wish to pick one of these as the basis of your Mathematical Exploration.

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☆ Do you remember the parent function of a Growth and Decay graph?

If you are given an initial amount when t = 0, this is the y-intercept, use the form: $y = ab^x$

If you are given the **PERCENT** increase or decrease and an initial amount use one of the following forms.

$$y = a(1+r)^t \quad \text{or} \quad y = a(1-r)^t$$

where: a is the initial amount
r is the percent expressed as a decimal
t is time

the growth factor is the quantity (1 + r)
the decay factor is the quantity (1 - r)

What if the interest is not compounded yearly?

Compound Interest: is interest paid on the initial investment (the principal), and on previously earned interest.

Simple Interest: is the interest paid only on the principal.

Compound Interest Formula

$$A = P(1 + r/n)^{nt}$$

A = the amount in the account
P = the initial principal deposited
r = annual rate (expressed as a decimal)
n = amount of times compounded per year
t = years in the account

Example:
You deposit 500 dollars into an account that pays 4.75% interest. Find the balance in the account after 8 years if the interest is compounded weekly.

Growth Application Formulas

$y = a(1+r)^t$	given a rate of increase usually yearly if time is in years.
$y = P\left(1+\frac{r}{n}\right)^{nt}$	used in banking with interest compounding n times a year.
$y = ab^t$	if b > 1
$y = Pe^{rt}$	used in banking with interest compounding continuously
$y = ae^{kt}$	used in applications like population growth which is continuous.
$y = ae^{-kt}$	used in science problems like radioactive decay where the decay is continuous

Decay Application Formulas

$y = a(1-r)^t$	given a rate of decrease usually yearly if time is in years.
$y = ab^t$	if 0 < b < 1
$y = ae^{-kt}$	used in science problems like radioactive decay where the decay is continuous

6. NUCLEAR POWER The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of k for this element?

Growth Application

The population, $A(t)$, in thousands, of a city is modeled by the function $A(t) = 30e^{(0.02)t}$ where t is the number of years after 2010. Use this model to answer these questions:

a What was the population of the city in 2010?

b By what percentage is the population of the city increasing each year?

c What will the population of the city be in 2020?

d When will the city's population be 60 000?

Exponential Decay Application

A casserole is removed from the oven and cools according to the model with equation $T(t) = 85e^{-0.1t}$, where t is the time in minutes and T is the temperature in °C.

a What is the temperature of the casserole when it is removed from the oven?

b If the temperature of the room is 25 °C, how long will it take for the casserole to reach room temperature?

An entomologist monitoring a grasshopper plague notices that the area affected by the grasshoppers is given by $A_n = 1000 \times 2^{0.2n}$ hectares, where n is the number of weeks after the initial observation.

a Find the original affected area.

b Find the affected area after: **i** 5 weeks **ii** 10 weeks.

c Find the affected area after 12 weeks.

d Draw the graph of A_n against n.

When a diesel-electric generator is switched off, the current dies away according to the formula $I(t) = 24 \times (0.25)^t$ amps, where t is the time in seconds.

a Find $I(t)$ when $t = 0$, 1, 2 and 3.

b What current flowed in the generator at the instant when it was switched off?

c Plot the graph of $I(t)$ for $t \geqslant 0$ using the information above.

d Use your graph or technology to find how long it takes for the current to reach 4 amps.

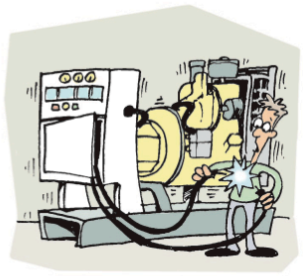
12 The weight of bacteria in a culture is given by $W(t) = 2e^{\frac{t}{2}}$ grams where t is the time in hours after the culture was set to grow.

a Find the weight of the culture when:

i $t = 0$ **ii** $t = 30$ min **iii** $t = 1\frac{1}{2}$ hours **iv** $t = 6$ hours.

b Use **a** to sketch the graph of $W(t) = 2e^{\frac{t}{2}}$.

13 The current flowing in an electrical circuit t seconds after it is switched off is given by $I(t) = 75e^{-0.15t}$ amps.



a What current is still flowing in the circuit after:

i $t = 1$ sec **ii** $t = 10$ sec?

b Use your graphics calculator to sketch $I(t) = 75e^{-0.15t}$ and $I = 1$.

c Find how long it will take for the current to fall to 1 amp.

Iryna has €5000 to invest in an account that pays 5.2% p.a. interest compounded annually. How long will it take for her investment to reach €20 000?

Homework

Chapter 4.8
4T: 1 - 5

Haese and Harris
Ch 4H: 2, 4, 6, 10, 14