Change of Base Formula

➔ Change of base formula:

$$\log_b a = \frac{\log_c a}{\log_c b}$$

This is in your booklet!

You will use this when you need to force a different base in order to evaluate a log or to change the log to any base!

*You will also need this if your GDC doesn't have the log shortcut key or ability to evaluate logs in any base.

Use the change of base formula to evaluate $\log_4 9$ to 3 significant figures.

Evaluate $\log_6 7$ to 3 significant figures

2 Given that $\log_3 x = y$, express $\log_9 x$ in terms of y.

Write $\log_3 x$ in $\log_9$

3 If $\log_a 2 = x$ and $\log_a 6 = y$, find in terms of x and y:

Write in $\log_a$

c $\log_2 36$

f $\log_2 3$

5 Given that $\log_4 a = b$ express y in terms of b.

Write in $\log_4$

d $y = \log_{\frac{1}{16}} \sqrt{a}$

Exponential and Logarithmic Equations

Solving exponential equations

We use logarithms to solve exponential equations

We already know how to solve exponential equations like $5^{3x} = 25^{x-2}$ because we can make the bases the same then set the exponents equal to each other.

But how do we solve exponential equations where we cannot make the bases equal??

$6e^{-0.4x} = 125$ $\quad 3^x = 7$ $\quad 5^{x-1} = 4^x$

$\left(\frac{1}{8}\right)^x = 9$ $\quad e^{-4x} = 52$ $\quad 7^{-x} = \frac{3}{5^{2x}}$

Examples:

Notice the bases cannot be written as the same base

Solve $5^x = 9$

To Solve:

1. Take the log(any base) of both sides
   Typically we use log or ln
2. Apply logarithm rules as needed
   Product, Quotient, Power, Change of Base
3. Solve for x
   If the question asks for an **exact value**, leave it.
   Otherwise use your calculator to find the value to 3 significant figures.

Examples:

Solve $6^x = 3^{x+1}$ giving your answer in the form $\frac{\ln a}{\ln b}$ where a and b are integers.

Note the form for your final answer

To Solve:
1. Take the ln of both sides
2. Use the Power Rule
3. Expand the brackets
4. Collect x-terms
5. Factorize and divide
6. Use Quotient Rule
7. Note the answer is in the required form! :)

Examples:

Solve $e^{3x} = 5^{1-x}$, giving an exact answer.

Note: requires an exact answer, no calculator!

To Solve:
1. Take the ln of both sides (because e is in the equation!)
2. Use the Power Rule
3. Expand brackets
4. Collect x-terms
5. Factor and divide
6. Leave as the **exact answer**

Examples:

Solve $3 \times 6^{x-1} = 2 \times 3^{x+2}$, giving your answer in the form $x = \frac{\ln a}{\ln b}$, where $a, b \in \mathbb{Z}$

Looking for an exact answer

To Solve:

1. Take the ln of both sides
2. Use Product Rule and Power Rule
3. Expand brackets
4. Collect x-terms
5. Use Product and Quotient Rules
6. Isolate x and make sure it is in the form required

Solving logarithmic equations

Two Methods:

1. Ensure that both sides of the equation contain only one logarithm that are in the same base, then equate the arguments

Make sure this is what your equation looks like before equating the arguments!

$$\log_a \_\_\_\_\_ = \log_a \_\_\_\_\_$$

Remember: The argument is the expression inside the brackets.

2. Solve using exponentials (convert to an exponential equation)

$$\log_a x = b \rightarrow a^b = x$$

Examples:

Solve $\log_a(x^2) = \log_a(3x+4)$

Note both sides of the equation are logs with the same base.

To Solve:

1. See there is only one log on each side
2. Equate the arguments
3. Solve for x
   See quadratic - think factor!
4. **CHECK** the solutions

Remember you cannot take the logarithm of a negative number **BUT** this does not mean x cannot be negative, you **MUST** check!

Examples:

Solve $\ln(12-x) = \ln x + \ln(x-5)$

Note both sides of the equation are ln (same base).

To Solve:

1. Use Product Rule to get a single ln on the right side
2. Equate the arguments
3. Solve for x
   See quadratic - think factor!
4. Check answer(s)

Examples:

Solve $\log_5(x-2)=3$

Note there is only one log in the equation.

To Solve:

1. Convert to an exponential equation
2. Solve for x
3. Check your solution(s)

Examples:

Solve $\log_2 x+\log_2(x-2)=3$

Note there are logs only on one side of in the equation.

To Solve:

1. Use Product Rule to combine logs
2. Convert to an exponential equation
3. Solve for x
4. Check your solution(s)

Homework

4O: 1, 3(a,b,d,e), 4, 5(a,b,c) -> 10-15 mins

4P: 2 -> 5-10 mins

4Q: 1(b,d,e), 2, 3 -> 10-15 mins

4R: 1 -> 5-10 mins

4S: 2 - 5 -> 10-15 mins