Arithmetic and Geometric Series

Remember: The sum of a sequence is called a ______________

Arithmetic Series: is the sum of consecutive terms of an arithmetic sequence.

Example

arithmetic sequence: 5, 12, 19, 26, 33, 40

arithmetic series: 5 + 12 + 19 + 26 + 33 + 40

When there are only a few terms, it is easy to find the sum of the series but when we have many terms such as 50 or 100, we need a formula to help us find the sum!

We need the formula when we can't use a calculator!

Let's find the formula for an arithmetic series:

Sum of the first n terms of a series: $S_n = u_1 + u_2 + u_3 + u_4 + u_5 + \cdots + u_n$

For an arithmetic series this would be:

$S_n = u_1 + \quad + \quad + \quad + \quad + \cdots +$

Now, reverse the order:

$S_n = u_n + \quad + \quad + \quad + \quad + \cdots +$

Adding these two equations for S_n vertically, term by term,

$$2S_n = (u_1 + u_n) + (u_1 + u_n) + (u_1 + u_n) + (u_1 + u_n) + (u_1 + u_n) + \cdots + (u_1 + u_n)$$

This is $(u_1 + u_n)$ added n times, so:

Dividing both sides by 2 gives:

Substitute $u_1 + (n-1)d$ for u_n, then

Example 2 **Find the sum of all positive odd integers less than 180.**

The series is $1 + 3 + 5 + \ldots + 179$.

Find n using the formula for the nth term of an arithmetic sequence.

Steps
1. Determine if arithmetic
2. What do you know?
3. Which formula is easier?
4. What info do you need and how will you find it?
5. Substitute and solve

Find S_n for each arithmetic series described.

$a_1 = 50, a_n = -50,$
$n = 15$

$a_1 = 42, n = 8, d = 6$

Example

Calculate the sum of the first 15 terms of the series
$29 + 21 + 13 + \ldots$

1. Decide which formula to used based of given info
2. Determine the value of d, u_1, and n
3. Substitute and solve

Example

a Find the number of terms in the series
$14 + 15.5 + 17 + 18.5 + \cdots + 50$

b Find the sum of the terms.

1. Decide which formula to use based on given info and what you want to solve
2. Determine the value of variables
3. Substitute and solve

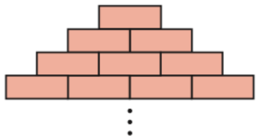
Example

a Write an expression for S_n, the sum of the first n terms, of the series $64 + 60 + 56 + \cdots$

b Hence, find the value of n for which $S_n = 0$

1. Decide which formula to use based of given info
2. Determine the value of variables
3. Substitute and solve

An arithmetic series has eleven terms. The first term is 6 and the last term is -27. Find the sum of the series.



A bricklayer builds a triangular wall with layers of bricks as shown. If the bricklayer uses 171 bricks, how many layers did he build?

Find the first two terms of an arithmetic sequence if the sixth term is 21 and the sum of the first seventeen terms is 0.

A geometric series is the ________ of the terms of a geometric sequence.

Let's derive the formula!

$$S_n = u_1 + u_1r + u_1r^2 + u_1r^3 + \cdots + u_1r^{n-2} + u_1r^{n-1}$$

multiply both sides by r.

Subtract the 1st equation from the 2nd.

Factorize both sides

Isolate the S_n

Finite Geometric Series Formulas!

→ You can find the sum of the first n terms of a geometric series using the formula:

$$S_n = \frac{u_1(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{u_1(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

these are in your booklet!

U_1 = first term
r = common ratio (2nd term divided by the 1st)
n = # of terms
S_n = Sum of the series

Where would you see an example of a Geometric Series in nature?

Koch snowflake

Calculate the sum of the first 12 terms of the series 1 + 3 + 9 + ⋯

Remember

$$S_n = \frac{u_1\left(r^n - 1\right)}{r - 1}$$

a Find the number of terms in the series $8192 + 6144 + 4608 + \cdots + 1458$.

b Calculate the sum of the terms.

For the geometric series $3 + 3\sqrt{2} + 6 + 6\sqrt{2} + \ldots$, determine the least value of n for which $S_n > 500$

*Use your GDC!

A geometric progression has first term of 0.4 and common ratio 2.
Find the value of n such that $S_n = 26\,214$

*Use your GDC!

Let S_n be the sum of an infinite geometric sequence such that $S_1 = 3$ and $S_2 = 4$.

a State the first term u_1.

b Calculate the common ratio r.

c Calculate u_5.

Each year a salesperson is paid a bonus of $2000 which is banked into the same account. It earns a fixed rate of interest of 6% p.a. with interest being paid annually. The total amount in the account at the end of each year is calculated as follows:

$$A_0 = 2000$$
$$A_1 = A_0 \times 1.06 + 2000$$
$$A_2 = A_1 \times 1.06 + 2000 \quad \text{and so on.}$$

a Show that $A_2 = 2000 + 2000 \times 1.06 + 2000 \times (1.06)^2$.

b Show that $A_3 = 2000[1 + 1.06 + (1.06)^2 + (1.06)^3]$.

c Find the total bank balance after 10 years, assuming there are no fees or charges.

Homework

Chapter 6.5

6G: 3, 4, 5

6H: 2, 4, 5

Chapter 6.6

6I: 1c, 2b, 3

6J: 2, 4, 5, 6 (all P2 questions)