

Warm-up: Some Review Problems

For $A(-1, 2, 5)$, $B(2, 0, 3)$ and $C(-3, 1, 0)$ find the position vector of:

- a A from O and the distance from O to A
- b C from A and the distance from A to C
- c B from C and the distance from C to B.

$$a) \vec{OA} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix} \quad |\vec{OA}| = \sqrt{30}$$

$$b) \vec{AC} = \begin{pmatrix} -2 \\ -1 \\ -5 \end{pmatrix} \quad |\vec{AC}| = \sqrt{30}$$

$$c) \vec{CB} = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix} \quad |\vec{CB}| = \sqrt{35}$$

A quadrilateral has vertices $A(1, 2, 3)$, $B(3, -3, 2)$, $C(7, -4, 5)$ and $D(5, 1, 6)$.

- Find \vec{AB} and \vec{DC} .
- What can be deduced about the quadrilateral ABCD?

$$\vec{AB} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$$

$$\vec{DC} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$$

$$\vec{AB} \parallel \vec{DC}$$

\therefore ABCD is a

$$|\vec{AB}| = |\vec{DC}|$$

parallelogram

- 4 The position vectors of A , B and C are given by $3\mathbf{i} + 4\mathbf{j}$, $x\mathbf{i}$, $\mathbf{i} - 2\mathbf{j}$ respectively. Find the value of x so that A , B and C are collinear and find the ratio $AB : BC$.

$$\vec{AC} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$$

$$\vec{AB} = \begin{pmatrix} x-3 \\ -4 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} \\ -4 \end{pmatrix}$$

$$k \begin{pmatrix} -2 \\ -6 \end{pmatrix} = \begin{pmatrix} x-3 \\ -4 \end{pmatrix}$$

$$k = \frac{2}{3}$$

$$-2k = x-3$$

$$-\frac{4}{3} = x-3$$

$$\boxed{\frac{5}{3} = x}$$

$$\vec{BC} = \begin{pmatrix} -\frac{1}{3} \\ 2 \end{pmatrix}$$

$$2:1$$

Unit Vectors

Have length 1 in a given direction

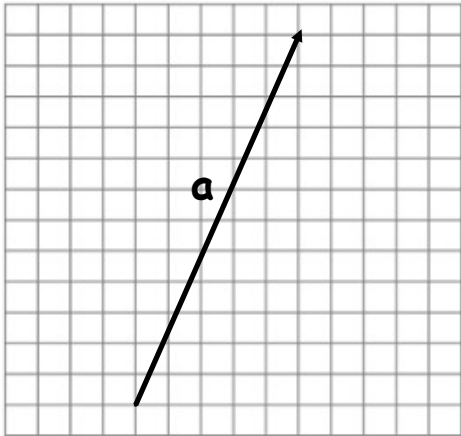
For example:

- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector as its length is $\sqrt{1^2 + 0^2} = 1$

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are 2-dimensional unit vectors in the positive x and y -directions respectively.

- $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ is a unit vector as its length is $\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = 1$

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are 3-dimensional unit vectors in the directions of the positive X , Y and Z -axes respectively.



What is vector \mathbf{a} ? $\vec{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

What is the length of \mathbf{a} ?

$$|\vec{a}| = 13$$

How can we make this length 1?

divide by 13

What do you think you have to do to the vector components to create a new vector with length 1?

$$\begin{pmatrix} \frac{5}{13} \\ \frac{12}{13} \end{pmatrix}$$

Conclusion:

To find a unit vector in the same direction as \mathbf{a}

1. Find the length: $|\mathbf{a}|$
2. Multiply vector \mathbf{a} by $\frac{1}{|\mathbf{a}|}$

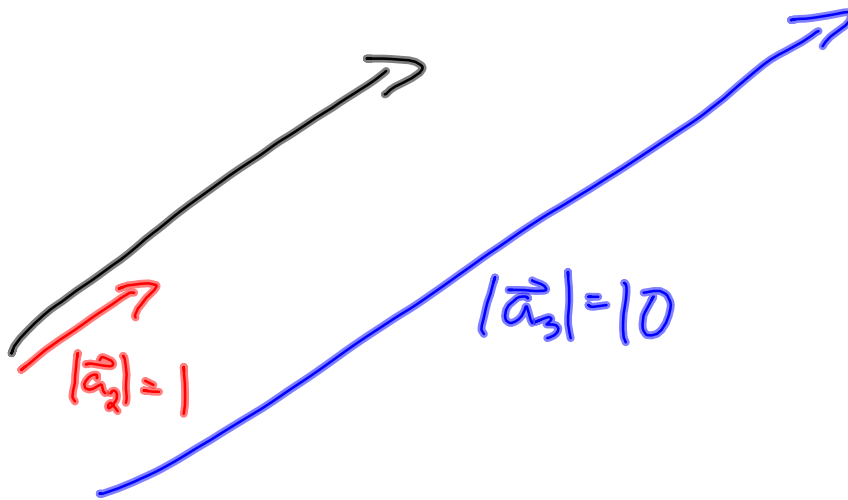
This results in a vector that is a scalar multiple of \mathbf{a} and has length 1.

→ A vector of length 1 in the direction of \mathbf{a} is found by using the formula $\frac{\mathbf{a}}{|\mathbf{a}|}$.

$$\frac{1}{|\vec{a}|} \vec{a}$$

We can use this to find a vector of ANY length
in the same direction as \mathbf{a} !

Think about it...if I can force the length of \mathbf{a} to be 1, then how
can I make it length 10?



Conclusion:

→ A vector of length k in the direction of \mathbf{a} is found by using the formula $k \frac{\mathbf{a}}{|\mathbf{a}|}$.

$$\frac{k}{|\vec{a}|} \vec{a}$$

- a** Find the unit vector in the same direction as the vector $3\mathbf{i} + 4\mathbf{j}$
- b** Find a vector of length 10 in the same direction as $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$$a) \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$$

$$b) \begin{pmatrix} \frac{30}{\sqrt{10}} \\ \frac{-10}{\sqrt{10}} \end{pmatrix}$$

Find k given that

$\begin{pmatrix} -\frac{1}{3} \\ k \end{pmatrix}$ is a unit vector.

$$\sqrt{\left(-\frac{1}{3}\right)^2 + k^2} = 1$$

$$\sqrt{\frac{1}{9} + k^2} = 1$$

$$\frac{1}{9} + k^2 = 1$$

$$k^2 = \frac{8}{9}$$

$$k = \pm \frac{\sqrt{8}}{3}$$

Find a vector \mathbf{b} of length 7 in the opposite direction to the vector $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

$$\mathbf{b} = \begin{pmatrix} -14 \\ 7 \\ 7 \end{pmatrix}$$

$$-\frac{7}{\sqrt{6}} \vec{a}$$

Find k for the unit vectors:

a $\begin{pmatrix} 0 \\ k \end{pmatrix}$

$$k = \pm 1$$

b $\begin{pmatrix} k \\ 0 \end{pmatrix}$

$$k = \pm 1$$

c $\begin{pmatrix} k \\ 1 \end{pmatrix}$

$$k = 0$$

d $\begin{pmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{pmatrix}$

$$k = \pm \frac{\sqrt{11}}{4}$$

e $\begin{pmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

$$k = \pm \frac{2}{3}$$

Find a vector **b** in:

- a** the same direction as $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and with length 3 units
- b** the opposite direction to $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ and with length 2 units
- c** the same direction as $\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ and with length 6 units
- d** the opposite direction to $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$ and with length 5 units.

a) $\begin{pmatrix} \frac{6}{\sqrt{5}} \\ -\frac{3}{\sqrt{5}} \end{pmatrix}$

b) $\begin{pmatrix} \frac{2}{\sqrt{17}} \\ \frac{8}{\sqrt{17}} \end{pmatrix}$

c) $\begin{pmatrix} \frac{-6}{\sqrt{18}} \\ \frac{24}{\sqrt{18}} \\ \frac{6}{\sqrt{18}} \end{pmatrix}$

d) $\begin{pmatrix} \frac{5}{\sqrt{10}} \\ \frac{10}{\sqrt{10}} \\ \frac{10}{\sqrt{10}} \end{pmatrix}$

Homework
Chapter 12.1
12F: 1-9