Warm-up: Some Review Problems

For $A(-1, 2, 5)$, $B(2, 0, 3)$ and $C(-3, 1, 0)$ find the position vector of:

a A from O and the distance from O to A

b C from A and the distance from A to C

c B from C and the distance from C to B.

a) $\vec{OA} = \begin{pmatrix} -1 \\ 2 \\ 5 \end{pmatrix}$ $|\vec{OA}| = \sqrt{30}$

b) $\vec{AC} = \begin{pmatrix} -2 \\ -1 \\ -5 \end{pmatrix}$ $|\vec{AC}| = \sqrt{30}$

c) $\vec{CB} = \begin{pmatrix} 5 \\ -1 \\ 3 \end{pmatrix}$ $|\vec{CB}| = \sqrt{35}$

A quadrilateral has vertices A(1, 2, 3), B(3, −3, 2), C(7, −4, 5) and D(5, 1, 6).

a Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

b What can be deduced about the quadrilateral ABCD?

a) $\vec{AB} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$ $\vec{DC} = \begin{pmatrix} 2 \\ -5 \\ -1 \end{pmatrix}$

b) $\vec{AB} \parallel \vec{DC} \rightarrow \vec{AB} = 1\,\vec{DC}$

$|\vec{AB}| = |\vec{DC}|$ Parallelogram

4 The position vectors of A, B and C are given by $3\mathbf{i} + 4\mathbf{j}$, $x\mathbf{i}$, $\mathbf{i} - 2\mathbf{j}$ respectively. Find the value of x so that A, B and C are collinear and find the ratio $AB : BC$.

$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\overrightarrow{OB} = \begin{pmatrix} x \\ 0 \end{pmatrix}$ $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} x-3 \\ -4 \end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$

$\begin{pmatrix} x-3 \\ -4 \end{pmatrix} = k\begin{pmatrix} -2 \\ -6 \end{pmatrix}$

$x-3 = -2k$ $-4 = -6k$

$x-3 = -2\left(\frac{2}{3}\right)$ $\frac{2}{3} = k$

$x-3 = \frac{-4}{3}$

$+3$ $+\frac{9}{3}$

$\boxed{x = \frac{5}{3}}$

$AB : BC$

$\begin{pmatrix} -\frac{4}{3} \\ -4 \end{pmatrix} : \begin{pmatrix} -\frac{2}{3} \\ -2 \end{pmatrix}$

$2 : 1$

Unit Vectors

Have length 1 in a given direction

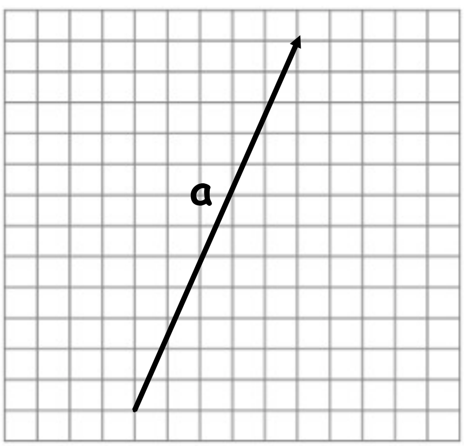
For example:

- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a unit vector as its length is $\sqrt{1^2 + 0^2} = 1$

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are 2-dimensional unit vectors in the positive x and y-directions respectively.

- $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$ is a unit vector as its length is $\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + 0^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = 1$

$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are 3-dimensional unit vectors in the directions of the positive X, Y and Z-axes respectively.



What is vector **a**? $\underline{a} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

What is the length of **a**?

13

How can we make this length 1?

divide by 13

What do you think you have to do to the vector components to create a new vector with length 1?

$$\begin{pmatrix} \frac{5}{13} \\ \frac{12}{13} \end{pmatrix} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = 1$$

Conclusion:

To find a unit vector in the same direction as **a**

1. Find the length: $|\mathbf{a}|$
2. Multiply vector **a** by $\frac{1}{|\mathbf{a}|}$

This results in a vector that is a scalar multiple of **a** and has length 1.

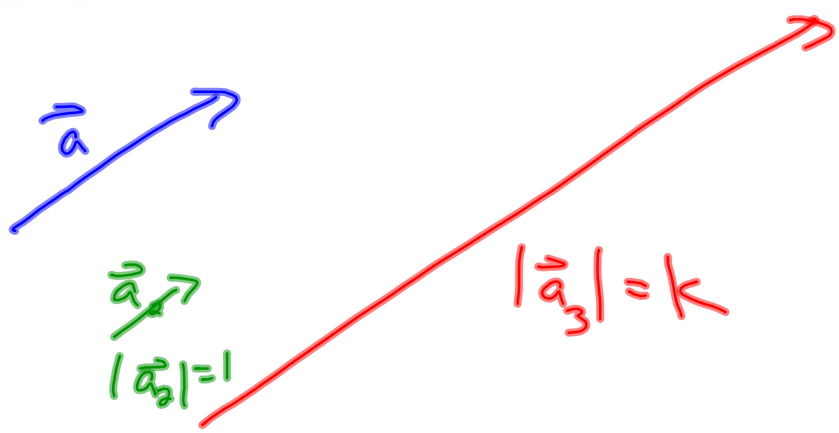
→ A vector of length 1 in the direction of **a** is found by using the formula $\frac{\mathbf{a}}{|\mathbf{a}|}$.

We can use this to find a vector of ANY length
in the same direction as **a**!

Think about it...if I can force the length of **a** to be 1, then how can I make it length 10?

Conclusion:

→ A vector of length k in the direction of $\mathbf{a}$ is found by using the formula $k\dfrac{\mathbf{a}}{|\mathbf{a}|}$.



a Find the unit vector in the same direction as the vector $3\mathbf{i} + 4\mathbf{j}$

b Find a vector of length 10 in the same direction as $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

a) $\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$

b) $\begin{pmatrix} \frac{30}{\sqrt{10}} \\ \frac{-10}{\sqrt{10}} \end{pmatrix}$

Find k given that $\begin{pmatrix} -\frac{1}{3} \\ k \end{pmatrix}$ is a unit vector.

$$k = \pm\sqrt{\frac{8}{9}}$$

$$\pm\frac{\sqrt{8}}{3}$$

$$\pm\frac{2\sqrt{2}}{3}$$

Find a vector **b** of length 7 in the opposite direction to the vector $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

$|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}$

$\vec{b} = \begin{pmatrix} \frac{-14}{\sqrt{6}} \\ \frac{7}{\sqrt{6}} \\ -\frac{7}{\sqrt{6}} \end{pmatrix}$

$7 \cdot \frac{-\vec{a}}{\sqrt{6}}$

$-\frac{7}{\sqrt{6}} \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Find k for the unit vectors:

a $\begin{pmatrix} 0 \\ k \end{pmatrix}$ **b** $\begin{pmatrix} k \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} k \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{pmatrix}$ **e** $\begin{pmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

$k = \pm 1$ $k = \pm 1$ $k = 0$ $k = \pm\sqrt{\frac{11}{16}}$ $k = \pm\frac{2}{3}$

Find a vector **b** in:

a the same direction as $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and with length 3 units

$\begin{pmatrix} \frac{6}{\sqrt{5}} \\ -\frac{3}{\sqrt{5}} \end{pmatrix}$

b the opposite direction to $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ and with length 2 units

$\begin{pmatrix} \frac{2}{\sqrt{17}} \\ \frac{8}{\sqrt{17}} \end{pmatrix}$

c the same direction as $\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ and with length 6 units

$\begin{pmatrix} -\frac{6}{\sqrt{18}} \\ \frac{24}{\sqrt{18}} \\ \frac{6}{\sqrt{18}} \end{pmatrix}$

d the opposite direction to $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$ and with length 5 units.

$\begin{pmatrix} \frac{5}{3} \\ \frac{10}{3} \\ \frac{10}{3} \end{pmatrix}$

Homework

Chapter 12.1

12F: 1-9