Warm-up: Some Review Problems

For $A(-1, 2, 5)$, $B(2, 0, 3)$ and $C(-3, 1, 0)$ find the position vector of:

a A from O and the distance from O to A

b C from A and the distance from A to C

c B from C and the distance from C to B.

A quadrilateral has vertices A(1, 2, 3), B(3, −3, 2), C(7, −4, 5) and D(5, 1, 6).

a Find $\overrightarrow{AB}$ and $\overrightarrow{DC}$.

b What can be deduced about the quadrilateral ABCD?

4 The position vectors of A, B and C are given by $3\mathbf{i} + 4\mathbf{j}$, $x\mathbf{i}$, $\mathbf{i} - 2\mathbf{j}$ respectively. Find the value of x so that A, B and C are collinear and find the ratio $AB : BC$.

Unit Vectors

Have length 1 in a given direction

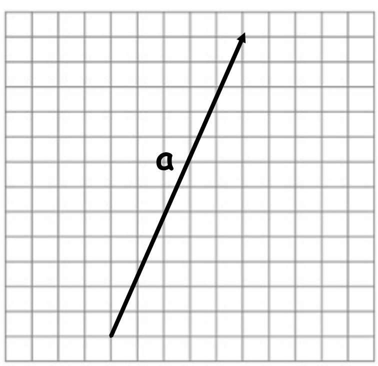
For example:

- $\begin{pmatrix}1\\0\end{pmatrix}$ is a unit vector as its length is $\sqrt{1^2+0^2}=1$

$\mathbf{i}=\begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{j}=\begin{pmatrix}0\\1\end{pmatrix}$ are 2-dimensional unit vectors in the positive x and y-directions respectively.

- $\begin{pmatrix}\frac{1}{\sqrt{2}}\\0\\-\frac{1}{\sqrt{2}}\end{pmatrix}$ is a unit vector as its length is $\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+0^2+\left(-\frac{1}{\sqrt{2}}\right)^2}=1$

$\mathbf{i}=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{j}=\begin{pmatrix}0\\1\\0\end{pmatrix}$ and $\mathbf{k}=\begin{pmatrix}0\\0\\1\end{pmatrix}$ are 3-dimensional unit vectors in the directions of the positive X, Y and Z-axes respectively.



What is vector **a**?

What is the length of **a**?

How can we make this length 1?

What do you think you have to do to the vector components to create a new vector with length 1?

Conclusion:

To find a unit vector in the same direction as **a**

1. Find the length: $|\mathbf{a}|$
2. Multiply vector **a** by $\frac{1}{|\mathbf{a}|}$

This results in a vector that is a scalar multiple of **a** and has length 1.

→ A vector of length 1 in the direction of **a** is found by using the formula $\frac{\mathbf{a}}{|\mathbf{a}|}$.

We can use this to find a vector of ANY length in the same direction as **a**!

Think about it...if I can force the length of **a** to be 1, then how can I make it length 10?

Conclusion:

→ A vector of length k in the direction of $\mathbf{a}$ is found by using the formula $k\frac{\mathbf{a}}{|\mathbf{a}|}$.

a Find the unit vector in the same direction as the vector $3\mathbf{i} + 4\mathbf{j}$

b Find a vector of length 10 in the same direction as $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Find k given that $\begin{pmatrix} -\frac{1}{3} \\ k \end{pmatrix}$ is a unit vector.

Find a vector $\mathbf{b}$ of length 7 in the opposite direction to the vector $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

Find k for the unit vectors:

a $\begin{pmatrix} 0 \\ k \end{pmatrix}$ **b** $\begin{pmatrix} k \\ 0 \end{pmatrix}$ **c** $\begin{pmatrix} k \\ 1 \end{pmatrix}$ **d** $\begin{pmatrix} -\frac{1}{2} \\ k \\ \frac{1}{4} \end{pmatrix}$ **e** $\begin{pmatrix} k \\ \frac{2}{3} \\ -\frac{1}{3} \end{pmatrix}$

Find a vector **b** in:

a the same direction as $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and with length 3 units

b the opposite direction to $\begin{pmatrix} -1 \\ -4 \end{pmatrix}$ and with length 2 units

c the same direction as $\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$ and with length 6 units

d the opposite direction to $\begin{pmatrix} -1 \\ -2 \\ -2 \end{pmatrix}$ and with length 5 units.

Homework

Chapter 12.1

12F: 1-9