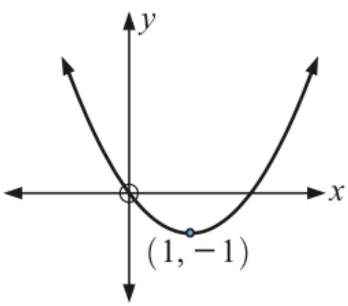
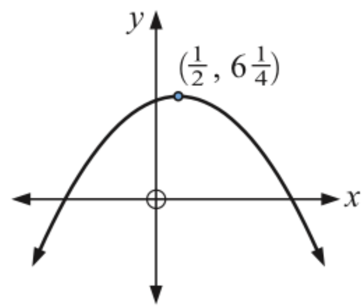
Warm-up

If $f(x) = 7 - 3x$, find in simplest form:

$f(a+3)$ $\qquad$ $f(x+2)$

For each of the following graphs, find the domain and range:





Perform the indicated operation.

$f(t) = t + 5$
$g(t) = t^2 - 4t$
Find $f(g(t))$

$f(t) = -3t - 3$
$g(t) = t^2 - 3t$
Find $f(g(t))$

Inverse Functions

→ The **inverse** of a function $f(x)$ is $f^{-1}(x)$. It reverses the action of that function.

Note that f^{-1} means the inverse of f; the '–1' is not an exponent (power).

The domain of f^{-1} is equal to the range of f.
The range of f^{-1} is equal to the domain of f.

★ Not all functions have an inverse.

$f^{-1}(x) \neq \frac{1}{f(x)}$

$(f(x))^{-1}$

When f and g are inverse functions, we write $g(x) = f^{-1}(x)$.

→ Functions $f(x)$ and $g(x)$ are inverses of one another if:

$(f \circ g)(x) = x$ for all of the x-values in the domain of g
$(g \circ f)(x) = x$ for all of the x-values in the domain of f.

The result of the composition of a function and its inverse is *x*

$$f\left(f^{-1}(x)\right) = x$$
$$f^{-1}\left(f(x)\right) = x$$

Show that **f(x) and g(x) are inverses of each other**

$$f(x) = 3x - 4 \text{ and } g(x) = \frac{x+4}{3}$$

$$f(g(x)) = \cancel{3}\left(\frac{x+4}{\cancel{3}}\right) - 4$$

$$= x + 4 - 4$$

$$= x$$

$$g(f(x)) = \frac{(3x - 4) + 4}{3}$$

$$= \frac{\cancel{3}x}{\cancel{3}}$$

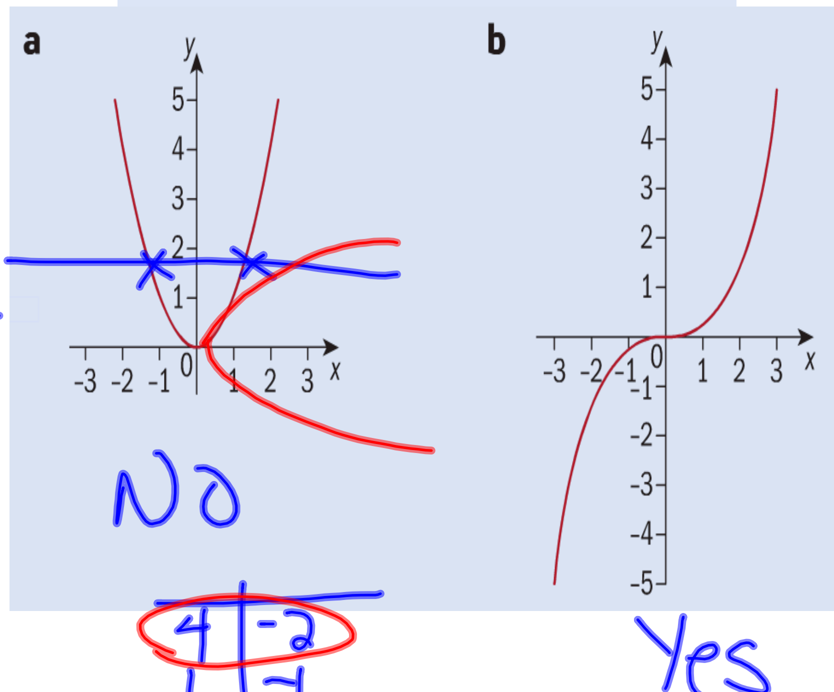
$$= x$$

The horizontal line test

→ You can use the **horizontal line test** to identity inverse functions.
If a horizontal line crosses the graph of a function more than once, there is no inverse function.

Example

Which of these functions have inverse functions?



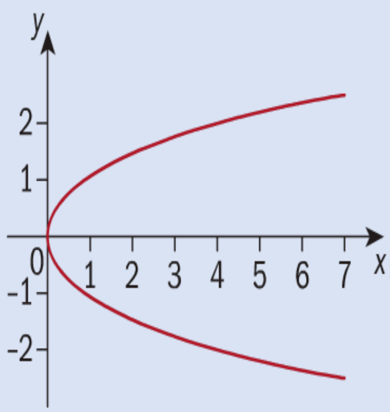
x	y
-2	4
1	1
0	0
1	1
2	4

4	-2
1	-1
0	0
1	1
4	2

Example

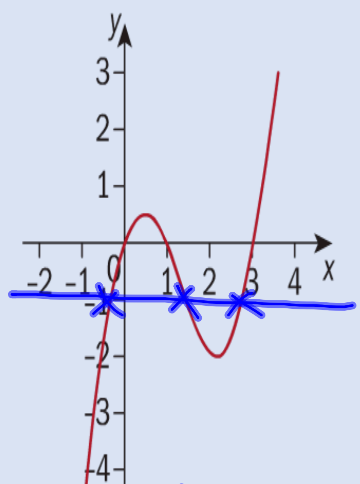
Which of these functions have inverse functions?

c



Yes

d

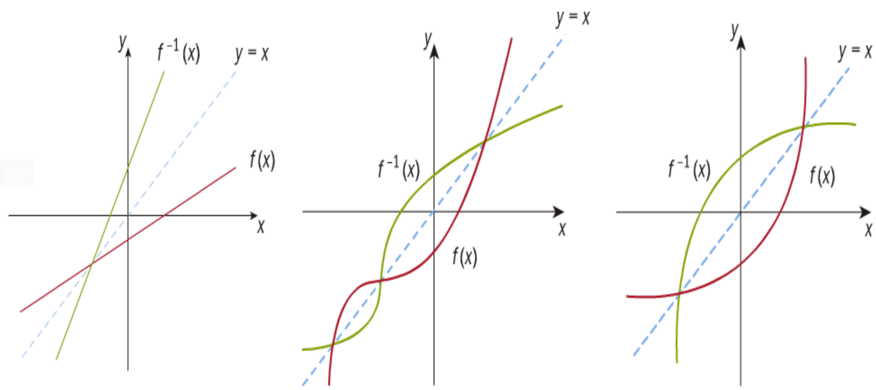


No

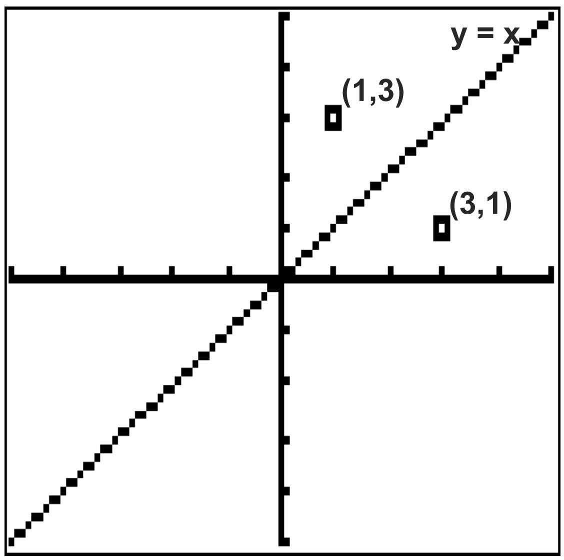
The graphs of inverse functions

→ The graph of the inverse of a function is a reflection of that function in the line $y = x$.

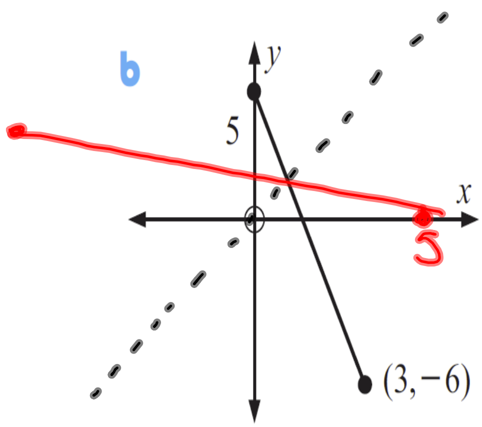
Here are some examples of functions and their inverse functions.

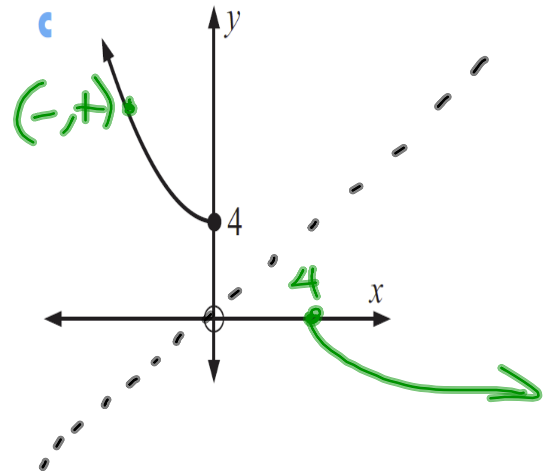


If (x, y) lies on the line $f(x)$, then (y, x) lies on $f^{-1}(x)$. Reflecting the function in the line $y = x$ 'swaps' x and y, so the point $(1, 3)$ reflected in the line $y = x$ becomes point $(3, 1)$.



Graph the inverse function





Solve the following equations for x:

$y = -3x - 7$ $\quad \frac{y+7}{-3} = x$

$y = \frac{2}{5}x + 4$ $\quad \frac{5}{2}(y-4) = x$

$\frac{5}{2}y - 10 = x$

$f(x) = -\frac{4}{3}x - 6$ $\quad -\frac{3}{4}(y+6) = x$

$-\frac{3}{4}y - \frac{9}{2} = x$

Finding inverse functions algebraically

Look at how the function $f(x) = 3x - 2$ is made up. We start with x on the left.

$$x \longrightarrow \boxed{\times 3} \longrightarrow \boxed{-2} \longrightarrow 3x - 2$$

To form the inverse function we reverse the process, using inverse operations.

$$\frac{x+2}{3} \longleftarrow \boxed{\div 3} \longleftarrow \boxed{+2} \longleftarrow x$$

$$\text{So } f^{-1}(x) = \frac{x+2}{3}$$

➔ To find the inverse function algebraically, replace $f(x)$ with y and solve for $\cancel{x}$

1.

Example If $f(x) = 4 - 3x$, find $f^{-1}(x)$.

y

2. swith x+y, solve for y

$$\frac{y-4}{-3} = x$$

$$f^{-1}(x) = \frac{x-4}{-3}$$

To check if the inverse function is correct, combine the functions!

Remember: $f\left(f^{-1}(x)\right) = x$

If $f(x) = \frac{x + 5}{4}$, find the inverse function $f^{-1}(x)$.

Replace f(x) with y.
Replace every x with y and every y with x.
Make y the subject.

Replace y with $f^{-1}(x)$.

$f^{-1}(x) = 4x - 5$

Find the inverse for each of these functions.

f $h(x) = 2x^3 + 3$

$$h^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

h $g(x) = \frac{2x}{5-x}, x \neq 5$

$$y = \frac{2x}{5-x}$$

$$y(5-x) = 2x$$

$$5y - xy = 2x$$

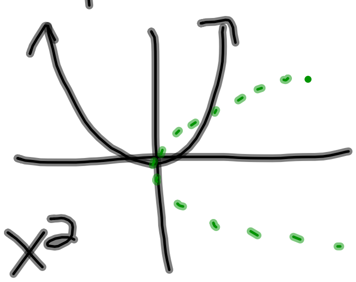
$$5y = 2x + xy$$

$$5y = x(2+y)$$

$$\frac{5y}{2+y} = x$$

$$g^{-1}(x) = \frac{5x}{2+x}$$

7 The function $f(x) = x^2$ has no inverse function. However, the square root function $g(x) = \sqrt{x}$ does have an inverse function. Find this inverse.
By comparing the range and domain explain why the inverse of $g(x) = \sqrt{x}$ is not the same as $f(x) = x^2$.



$|x|^2$

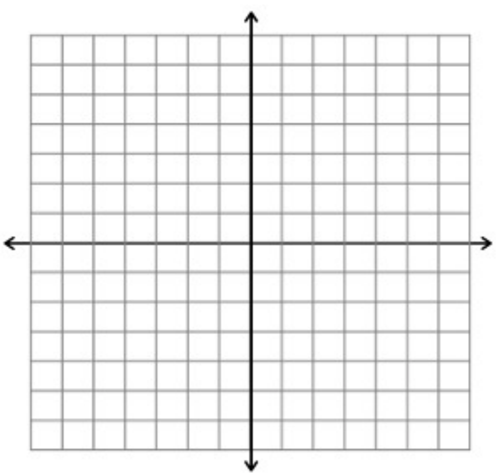
$g^{-1}(x) = x^2, \underbrace{x \geq 0}_{\text{Restrict the Domain}}$

Consider $f : x \mapsto 2x + 3$.

a On the same axes, graph f and its inverse function f^{-1}.

b Find $f^{-1}(x)$ using:
  i coordinate geometry and the gradient of $f^{-1}(x)$ from a
  ii variable interchange.

c Check that $(f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x$



Homework

Chapter 1.5

1G: 1, 2

1H: 1 - 6

Need extra practice?

Haese and Harris Chapter 1H, pg 64
1, 2, 3(a,c,d,e), 4, 7b, 9, 10, 12, 14