Fast Five - Investigation

Investigation – perfect square trinomials

Solve these equations by factorization.

1 $x^2 + 10x + 25 = 0$ **2** $x^2 + 6x + 9 = 0$

3 $x^2 + 14x + 49 = 0$ **4** $x^2 - 8x + 16 = 0$

5 $x^2 - 18x + 81 = 0$ **6** $x^2 - 20x + 100 = 0$

What do you notice? Describe any patterns you see in the original quadratic equations.

A trinomial is a polynomial with three terms.
Why do you think these are called 'perfect square trinomials'?

Solving Quadratic Equations ~ Day 2

Today we will learn how to solve a quadratic equation by:

Completing the Square

Pattern:	Examples:
$a^2 + 2ab + b^2 = (a + b)^2$	$x^2 + 14x + 49 = (x + 7)^2$
$a^2 - 2ab + b^2 = (a - b)^2$	$x^2 - 14x + 49 = (x - 7)^2$

We will "force" a perfect square trinomial to solve a quadratic equation. This method is called *completing the square*.

Steps for completing the square

Key Concept — *Completing the Square*

- **Words** To complete the square for any quadratic expression of the form $x^2 + bx$, follow the steps below.

  Step 1 Find one half of b, the coefficient of x.

  Step 2 Square the result in Step 1.

  Step 3 Add the result of Step 2 to $x^2 + bx$.

- **Symbols** $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$

Rules:
1. You must keep the equation balanced.
If you add something to the left side, you must add it to the right.
-or-
If you add something, you must also subtract it.

2. The leading coefficient of the x^2 term MUST be 1, before completing the square.

Find the value of c that makes each trinomial a perfect square. Then write the trinomial as a perfect square.

9. $x^2 + 10x + c$

10. $x^2 - 14x + c$

11. $x^2 + 24x + c$

12. $x^2 + 5x + c$

13. $x^2 - 9x + c$

14. $x^2 - x + c$

A few more:

$x^2 - \frac{1}{2}x + c$ $\quad$ $x^2 - \frac{4}{5}x + c$ $\quad$ $x^2 - 2.4x + c$

Square Root Property: used to eliminate an exponent of 2.

Key Concept — **Square Root Property**

For any real number n, if $x^2 = n$, then $x = \pm\sqrt{n}$.

Therefore, when you take the square root of an expression, you will have 2 answers! positive and negative.

Let's try solving the equation by completing the square.

$$x^2 + 8x + 39 = 0$$

$$x^2 + 8x + \quad = -39$$

$$x^2 + 8x + \_\_\_\_ = -39 \_\_\_\_$$

Steps

1. First subtract the constant term from both sides.
2. Then add to both sides the value you need to complete the square.
3. Factor the left side and simplify the right.
4. Now finish by using the square root property.

Examples

$x^2 - 6x + 5 = 0$

$x^2 + 2x + 10 = 0$

$x^2 + 2x - 4 = 0$

$x^2 + 10x + 80 = 0$

If **a** ≠ **1** you have two options:

Factor **a** out and continue completing the square (make sure you put brackets around EVERYTHING!)

$-3x^2 + 12x + 5 = 0$

$-3\left[x^2 - 4x - \frac{5}{3} = 0\right]$

$-3\left[x^2 - 4x + \quad = \frac{5}{3} + \quad\right]$

Divide EVERY term by a and continue completing the square. (You will have to deal with fractions)

$-3x^2 + 12x + 5 = 0$

$x^2 - 4x - \frac{5}{3} = 0$

$x^2 - 4x + \quad = \frac{5}{3} + \quad$

Solve each equation by completing the square.

a $2x^2 + 8x = 6$ **b** $3x^2 - 15x = 2$

Write $y = x^2 + 4x + 3$ in the form $y = (x-h)^2 + k$

Homework

Chapter 2: 2C and 2D

Need more practice?
Haese and Harris Chapter 6 A.2
Chapter 6 A.2 Worksheet