Solving Exponential Equations

Exponential Equation: is an equation where the unknown variable is the exponent.

You can write an exponential equation in the form: $a^x = b^y$.

Example

Solve $3^{x-1} = 3^{5x}$

$x - 1 = 5x$

$-1 = 4 \cdot x$

$-\frac{1}{4} = x$

Steps:

1. If the bases are the same, you may set the exponents equal to one another.
3. Solve the new equations from the exponents for x.
4. Check your answer into the original equation.

Solve $3^{3x+1} = 81$

$$3^{3x+1} = 3^4$$

$$3x+1 = 4$$

$$3x = 3$$

$$\boxed{x = 1}$$

Steps:

1. Make the bases match (change the 81 to a base of 3 with an exponent of what?)
2. **Once the bases are the same**, you may set the exponents equal to one another.
3. Solve the new equation of the exponents for x.
4. Check your answer (plug into the orginial equation).

Note: Always make sure there is only ONE base on each side of the equal sign and that they match.

Solve for x

a $2^x = 32$

$x = 5$

b $3^{1-2x} = 243$

$x = -2$

c $3^{x^2-2x} = 27$

$x = 3, -1$

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

d $5^{2x-1} - 25 = 0$

$x = \frac{3}{2}$

e $7^{1-x} = \frac{1}{49}$

$x = 3$

Solve for x

$$3x^{-\frac{3}{5}} = 24$$

$$x^{-\frac{3}{5}} = 8$$

$$\left(x^{-\frac{3}{5}}\right)^{-\frac{5}{3}} = \left(2^{3}\right)^{-\frac{5}{3}}$$

$$x = 2^{-5}$$

$$x = \frac{1}{32}$$

Solve for x

1) $a^{-\frac{1}{6}} = \left(\frac{1}{2}\right)^{-6} = 2^{6}$

$a = 64$

2) $b^{\frac{3}{2}} = 27$ $\quad (3^3)^{\frac{2}{3}}$

$b = 9$

3) $-1458 = -2x^{\frac{3}{2}}$

$x = 81$

4) $-4000 = -4(4x)^{\frac{3}{2}}$

$x = 25$

5) $8 = (16m)^{\frac{3}{5}}$

$m = 2$

6) $x^{\frac{3}{2}} - 6 = 723$

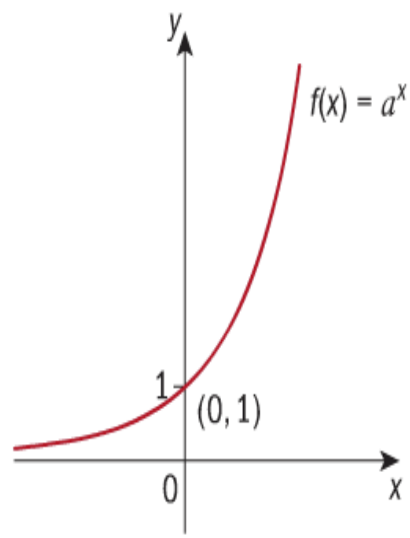
$x = 81$

Graphs and properties of exponential functions

→ An **exponential function** is a function of the form

$$f(x) = a^x$$

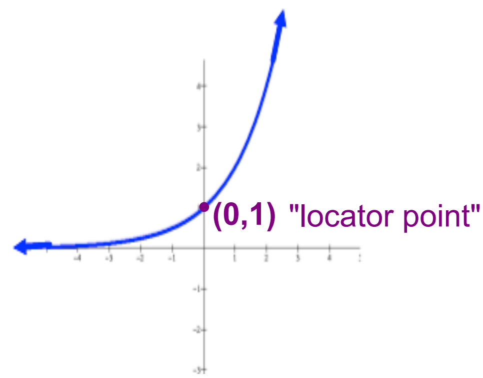
where a is a positive real number (that is, $a > 0$) and $a \neq 1$.



Characteristics of Growth

- x - axis is an asymptote of the graph!
- the graph rises from left to right.
- the graph passes through the points $(0,1)$, $(1,a)$, and $\left(-1,\frac{1}{a}\right)$.
- Domain: all real numbers
- Range: $y > 0$
- The function is continuous.

Exponential Growth



as $x \to -\infty$ $y \to 0$

Note!!!

The Horizontal Asymptote (HA): $y = 0$

If the graph has a <u>vertical transformation</u>, it will be $y = \#$

Desmos Activity

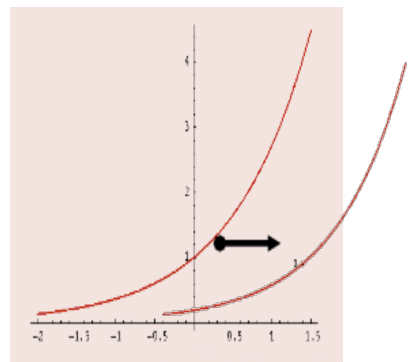
Marbleslides: Exponentials

1. Go to student.desmos.com
2. Sign in (use your Google account)
3. Enter the class code: scc9
4. Begin and have fun!

Transformations of Exponential Functions

$y=a(b)^{x-h} + k$

The h causes the graph to shift horizontally



Does a horizontal shift affect the Domain? NO

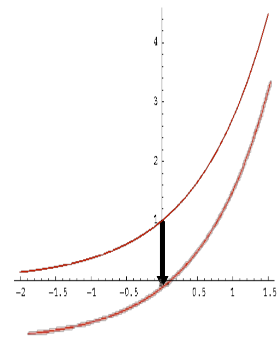
Does a horizontal shift affect the Range? NO

Does a horizontal shift affect the HA? NO

Does a horizontal shift affect the y-intercept? YES

Transformations of Exponential Functions

$y=a(b)^{x-h} + k$ The k causes the graph to shift up or down



Does a horizontal shift affect the Domain? No

Does a horizontal shift affect the Range? Yes

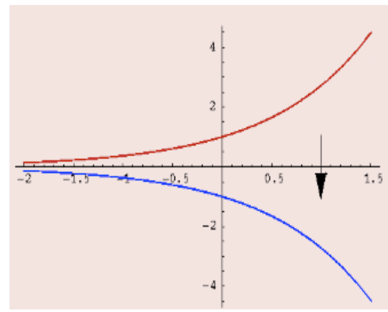
Does a horizontal shift affect the HA? Yes

Does a horizontal shift affect the y-intercept? Yes

Transformations of Exponential Functions

$y=a(b)^{x-h} + k$

When $a < 0$, the graph will reflect ("flip") across the x-axis



$y=-a(b)^x$

When **a** is negative, the graph will flip across the x-axis.
The inequality sign for the RANGE will flip as well:
y < the value of the horizontal asymptote

Exponential Growth Example

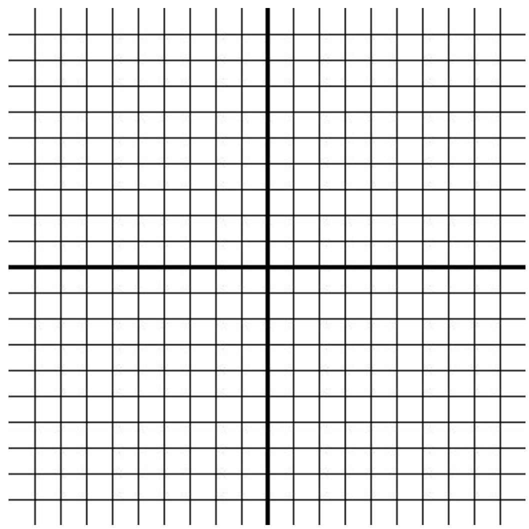
Graph: $y = 2^x$

Characteristics:

1. Locator point:___________
2. End Behavior:___________
3. Horizontal asymptote:_______
4. Domain:_____________
5. Range:______________

Steps

1. Make a table of values.
2. Plot the points.
3. Draw the graph left to right.



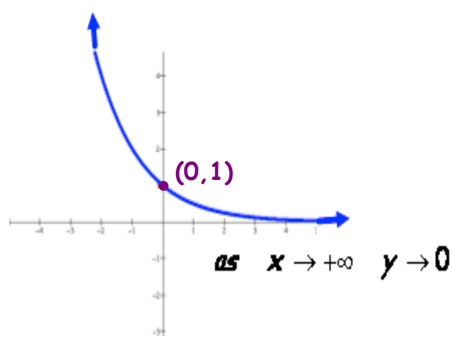
Exponential Decay Function:
the function has a negative slope and a > 0 and 0 < b < 1.

$y = a*b^x$; a > 0 and 0 < b < 1

Characteristics of Decay

- x - axis is an asymptote of the graph!
- the graph falls from left to right.
- the graph passes through the points (0,1) and (1,b).
- Domain: all real numbers
- Range: y > 0
- The function is continuous

Exponential Decay

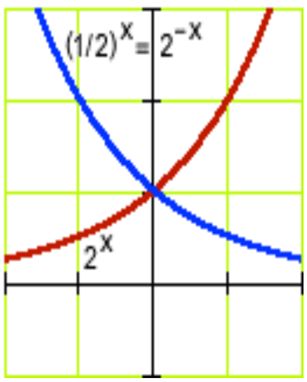


Note: $y=3^{-x}$ and $y=\frac{1}{3}^x$ result in the same graph. Why?

The Parent Function

No Transformations b > 1

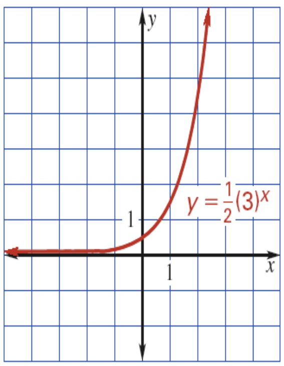
$y=a(b)^x$



- Vertical Shifts
- Horizontal Shifts
- and Reflections!

Note: $y = \left(\frac{1}{2}\right)^x = 2^{-x}$ **What type of transformation occurs?**

Classify each function as an exponential growth function or an exponential decay function.

1


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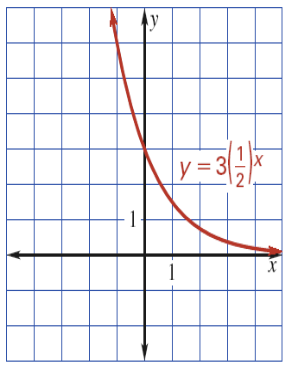

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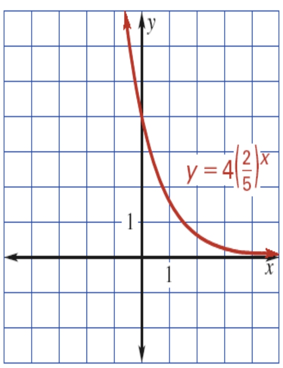

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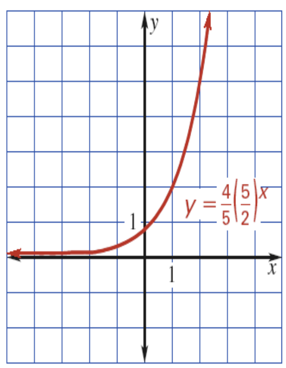
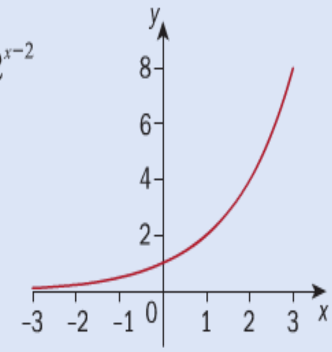


The diagram shows the sketch of $f(x) = 2^x$
On the same axes sketch the graph of $g(x) = 2^{x-2}$



The natural exponential function

Exponential functions with base e is the most common!

What is e?

- approximately 2.71828
- irrational number
- found in nature
- as a function, it is called the natural exponential function

Mathematics sometimes throws out some surprising and beautiful results.

Here is one such result.

To 20 decimal places e = 2.718 281 828 459 045 235 36...

There is no obvious pattern to this chain of numbers.

However look at this series, which gives a value of e:

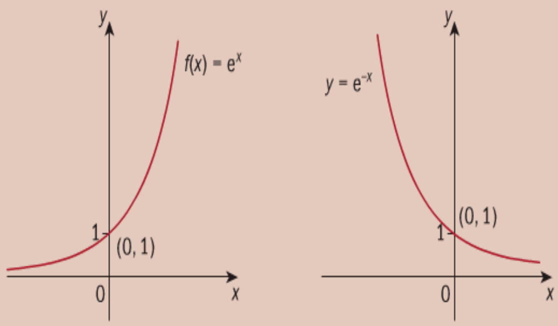
$$e = 1 + \frac{1}{1} + \frac{1}{2\times1} + \frac{1}{3\times2\times1} + \frac{1}{4\times3\times2\times1} + \frac{1}{5\times4\times3\times2\times1} + \dots$$

You might wonder about the connection between this series and the value of e.

[See the Theory of Knowledge page at the end of this chapter for thoughts and discussion on beauty in mathematics.]

Graphing

→ The graph of the exponential function $f(x) = e^x$ is a graph of exponential growth and the graph of $f(x) = e^{-x}$ is a graph of exponential decay.



Transformations are the same

$y=a(b)^{x-h} + k$

$y=a(e)^{x-h} + k$

Homework:

Chapter 4.2

4D: 1(pick 3), 2(c,d), 3

4E: 1(pick 3), 2(pick 3), 3(b,c)

Chapter 4.3

4F: All (given WS with graphs)