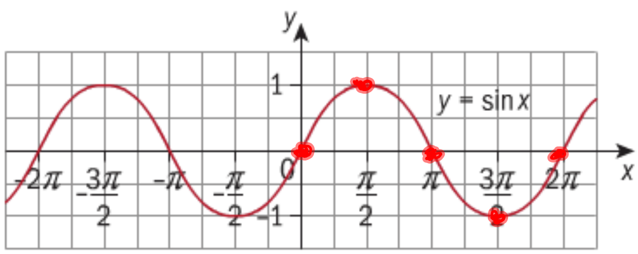
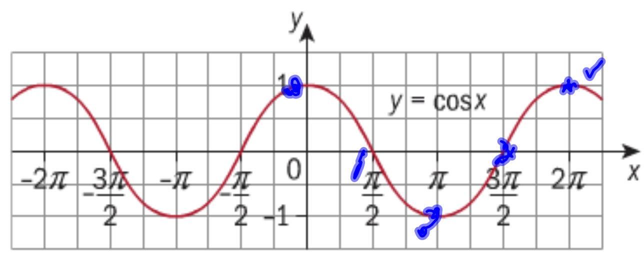
The Basics of Sine and Cosine Curves

You need to be very familiar with the features of the basic sine and cosine curves.

Period - how long it takes to complete one full cycle

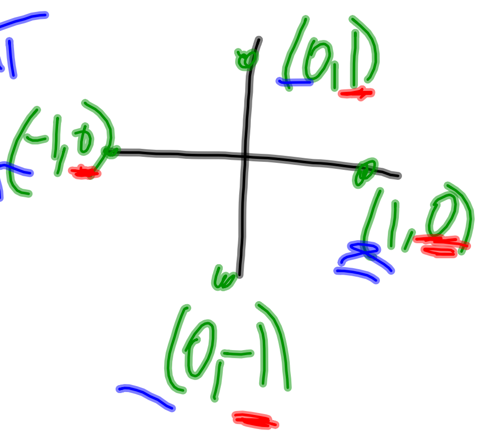
Amplitude - distance from axis of the curve (mean line) to max/min





Period: 2π

Amplitude: 1



Period: 2π

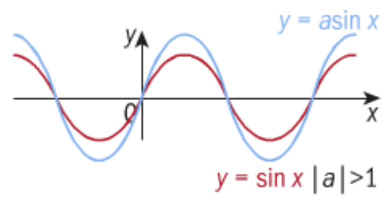
Amplitude: 1

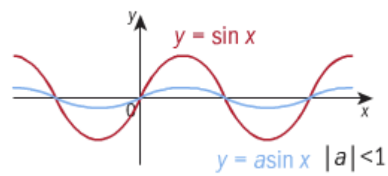
THE FAMILY $y = a\sin x$

What did you notice from the investigation?

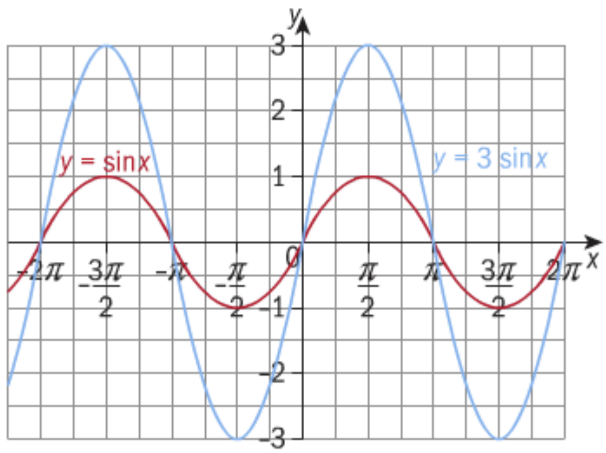
→ The functions $y = a\sin x$ and $y = a\cos x$ are **vertical stretches** of the sine and cosine functions.
When the graph of a function undergoes a vertical stretch, every y-value in the original function is multiplied by the value of a.
If $|a| > 1$, the function will appear to stretch away from the x-axis.

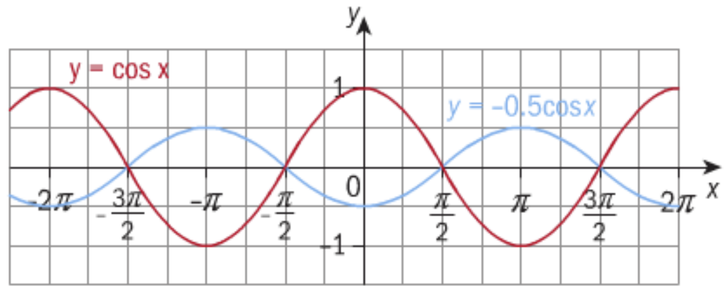
If $0 < |a| < 1$, the function will appear to compress closer to the x-axis.
If a is negative, the function will also be reflected over the x-axis.
With a vertical stretch, the **amplitude** of the sine or cosine function will change from 1 to $|a|$. The period of the function will not change.



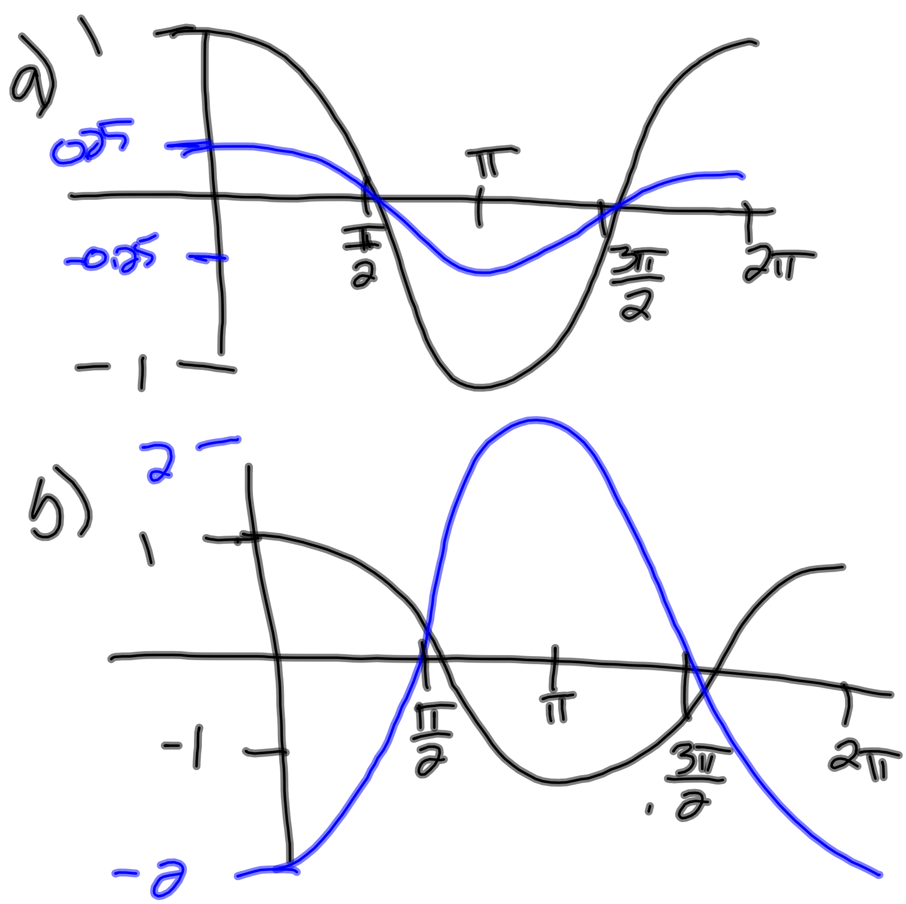


Examples of functions in the form y = a sinx or y = a cosx





Sketch the graph of $y = \cos x$.
On the same set of axes, sketch the graph of:
a $y = 0.25\cos x$ **b** $y = -2\cos x$



THE FAMILY $y = \sin bx, \quad b > 0$

What did you notice from the investigation?

Per = $\frac{2\pi}{b}$

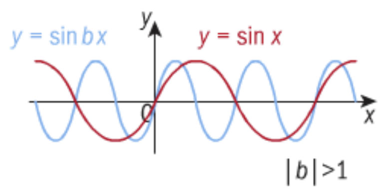
➔ The functions $y = \sin(bx)$, $y = \cos(bx)$ and $y = \tan(bx)$ represent horizontal stretches of the sine, cosine and tangent functions. When the graph of a function undergoes a horizontal stretch, every x-value in the original function is multiplied by $\frac{1}{b}$.

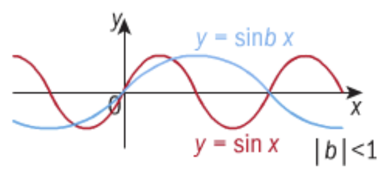
We could also say that every x-value in the original function is *divided* by b.

Multiplying (or dividing) the x-values by a number in this way changes the **period** of a trigonometric function.

- If $|b| > 1$, the period will be shorter, and the function will appear to compress toward the y-axis.
- If $0 < |b| < 1$, the period will be longer, and the function will appear to stretch away from the y-axis.
- If b is negative, the function will also be reflected over the y-axis.

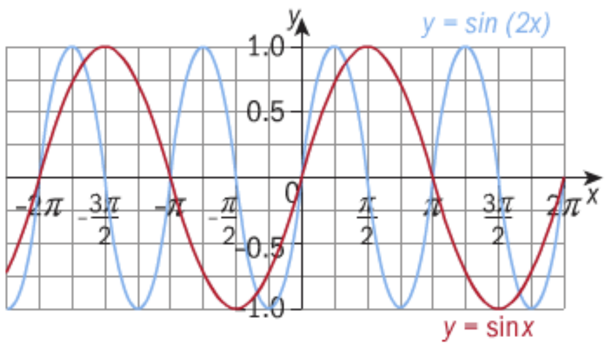
When a sine or cosine function undergoes a horizontal stretch, the period of the function will change from 2π to $\frac{2\pi}{|b|}$, or from 360° to $\frac{360°}{|b|}$.



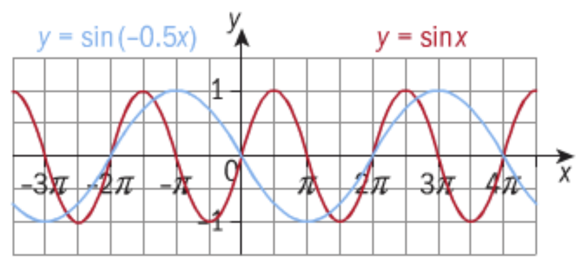


Examples of functions in the form y = sin bx

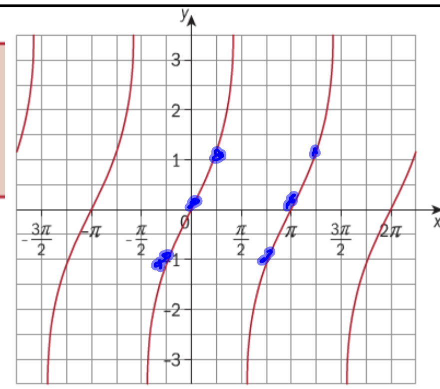
- In the graph below, the sine curve in blue has a period of π.



- In the graph below, the sine curve in blue has a period of 4π. The function has also been reflected about the **y**-axis.

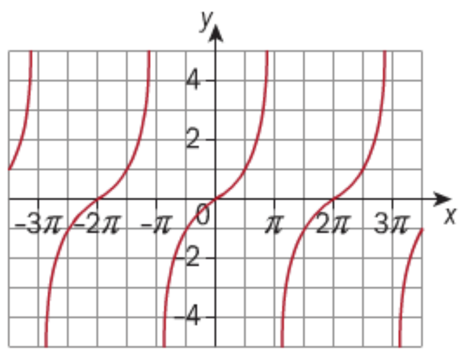


→ For a function in the form $y = \tan(bx)$, the period will change from π to $\frac{\pi}{|b|}$, or from 180° to $\frac{180°}{|b|}$.



Example of y = tan bx

$y = \tan(0.5x)$

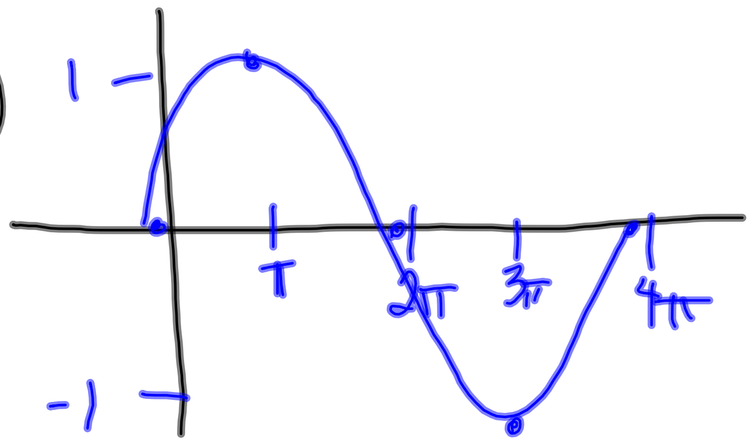


$Per = \frac{\pi}{\frac{1}{2}}$

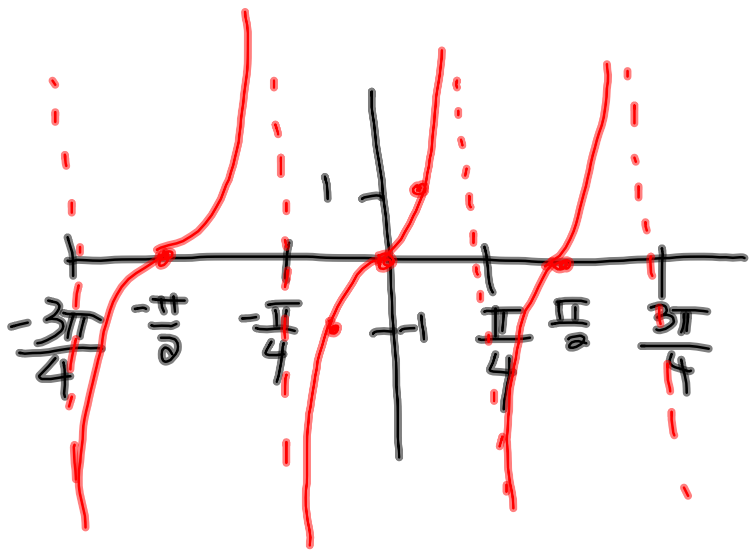
$= 2\pi$

Sketch the graph of: **a** $y = \sin(0.5x)$ **b** $y = \tan(2x)$ **c** $y = 2\cos(3x)$

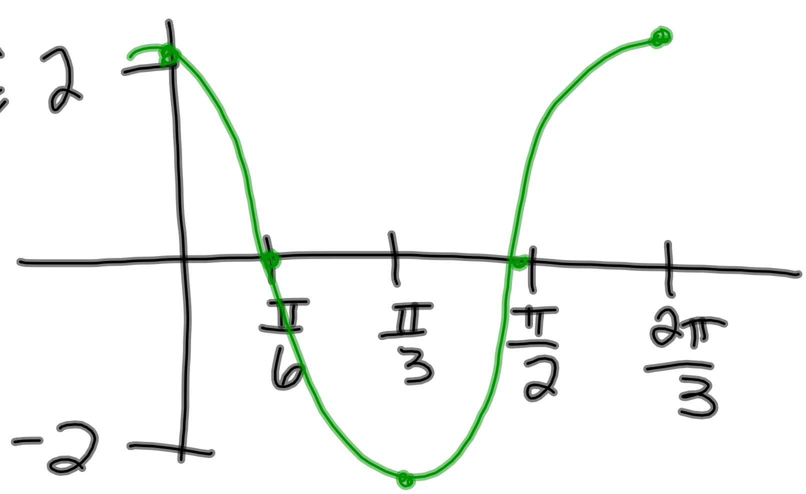
a)



b)



c)



THE FAMILIES $y = \sin(x - c)$ AND $y = \sin x + d$

What did you notice from the investigation?

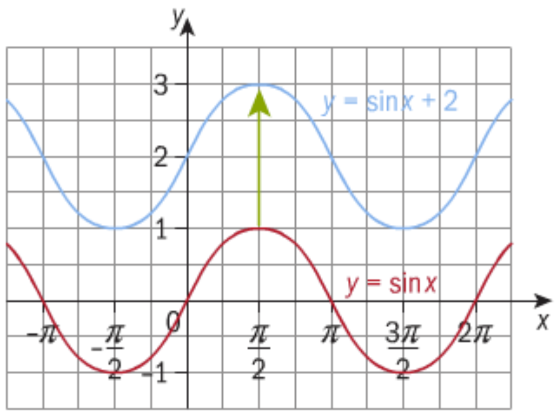
→ The function $y = \sin(x) + d$ is a **vertical translation** of the standard sine curve.
The curve shifts up if d is positive, down if d is negative.

The function $y = \sin(x - c)$ is a **horizontal translation** of the standard cosine curve. The curve shifts to the right if c is positive, left if c is negative.

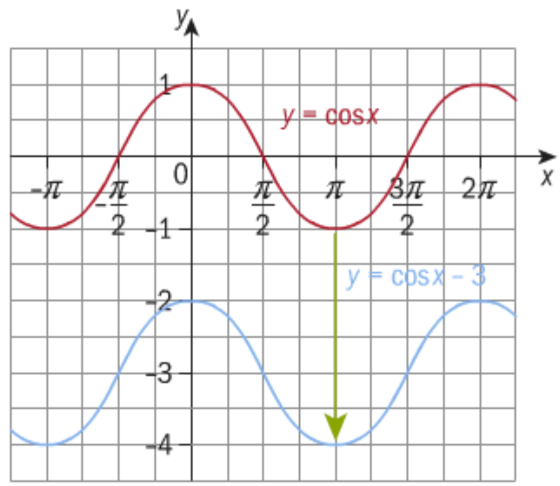
A horizontal translation is also known as a 'phase shift'.

Examples of functions in the form y = sinx + d and y = cosx + d

This graph shows a vertical translation. The sine curve has been shifted up 2 units. The green arrow shows the direction of the translation.

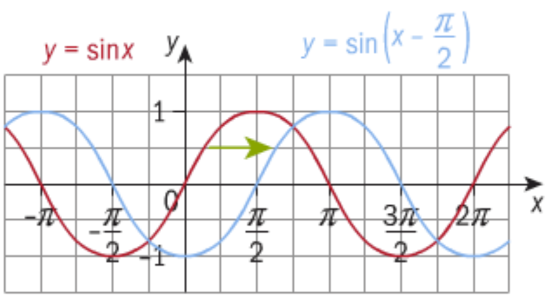


This graph shows a vertical translation. The cosine curve has been shifted down 3 units. The green arrow shows the direction of the translation.

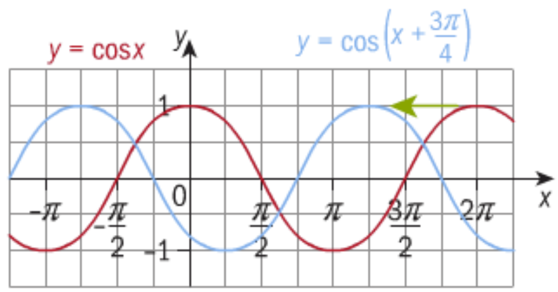


Examples of functions in the form y = sin(x-c) and y = cos(x-c)

- This graph shows a horizontal translation. The sine curve has been shifted $\frac{\pi}{2}$ units to the right. The green arrow shows the direction of the translation.



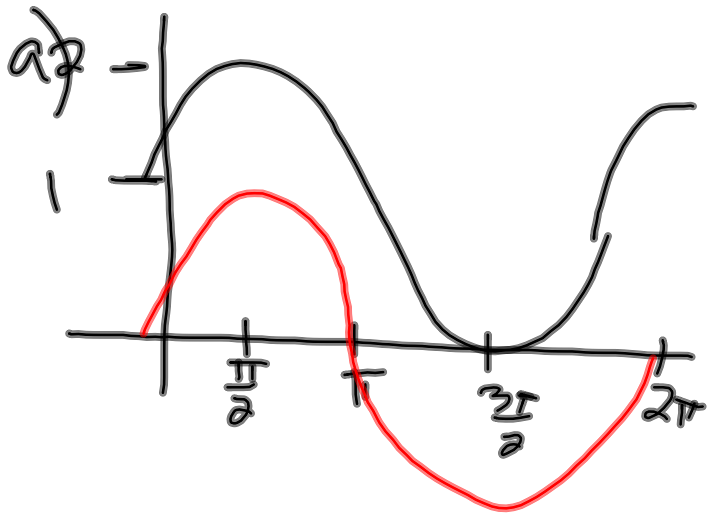
- The graph below shows a horizontal translation. The cosine curve has been shifted $\frac{3\pi}{4}$ units to the left.

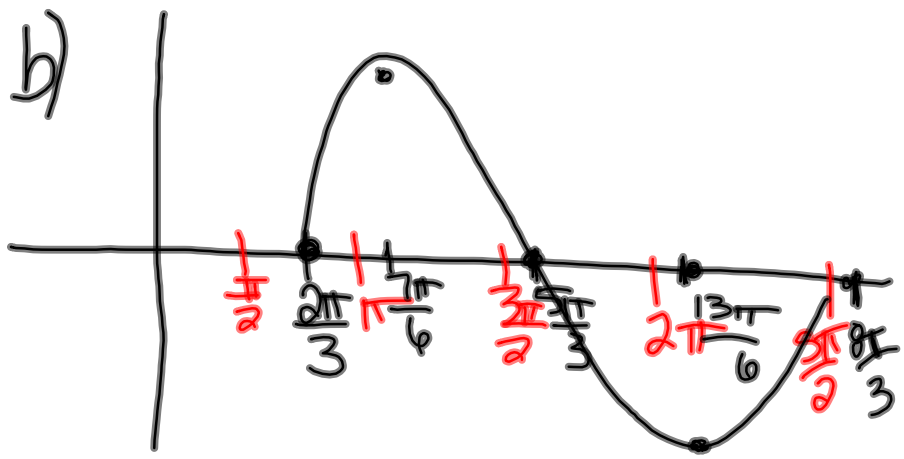


Sketch the graph of $y = \sin x$.

On the same set of axes, sketch the graph of:

a $y = \sin x + 1$ **b** $y = \sin\left(x - \frac{2\pi}{3}\right)$ **c** $y = \sin\left(x - \frac{2\pi}{3}\right) + 1$

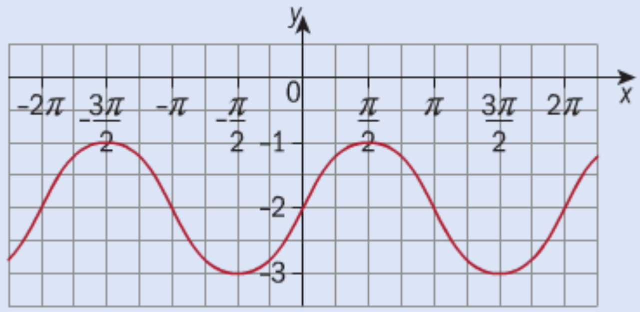




$\frac{\pi}{2} + \frac{2\pi}{3} = \frac{7\pi}{6}$ $\pi + \frac{2\pi}{3} = \frac{5\pi}{3}$

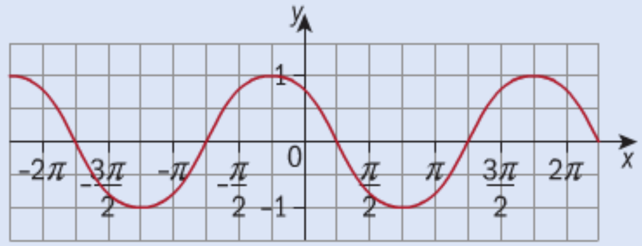
$\frac{3\pi}{2} + \frac{2\pi}{3} = \frac{13\pi}{6}$ $2\pi + \frac{2\pi}{3} = \frac{8\pi}{3}$

a Write a sine equation.



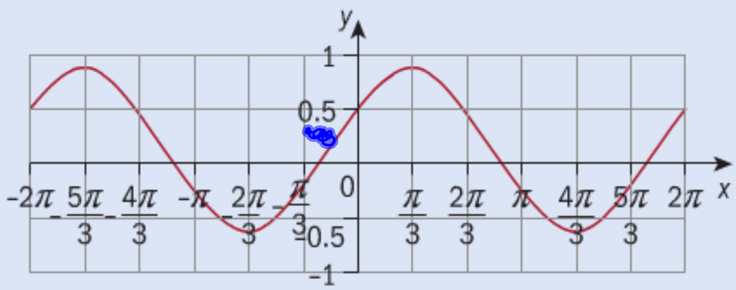
$y = \sin x - 2$

b Write a cosine equation.



$y = \cos\left(x + \frac{\pi}{4}\right)$

c Write one sine and one cosine equation.



$$y = 0.5\cos\left(x - \frac{\pi}{3}\right) + 0.25$$

$$y = 0.5\sin\left(x + \frac{\pi}{6}\right) + 0.25$$

Without using technology, sketch the graphs of:

a $y = 2\sin x$ **b** $y = -2\sin x$ for $0 \leqslant x \leqslant 2\pi$.

Without using technology, sketch the graph of $y = \sin 2x$ for $0 \leqslant x \leqslant 2\pi$.

On the same set of axes graph $y = \cos x$ and $y = \cos\left(x - \frac{\pi}{3}\right)$.

Without using technology, sketch the graph of $y = 2\cos 2x$ for $0 \leqslant x \leqslant 2\pi$.

Without using technology, sketch the graph of $y = \tan(x + \frac{\pi}{4})$ for $0 \leqslant x \leqslant 3\pi$.

Without using technology, sketch the graph of $y = \tan 2x$ for $-\pi \leqslant 0 \leqslant \pi$.

Homework

Chapter 13.5

13I: 1-12

13J: 1-12