Trigonometric Identities and Relationships

From your booklet...

Trigonometric identity	$\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean identity	$\cos^2\theta + \sin^2\theta = 1$
Double angle formulae	$\sin 2\theta = 2\sin\theta\cos\theta$
	$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

You may want/need to use rearrangements of these formulas such as:

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

Simplify: a $3\cos\theta + 4\cos\theta$ b $\sin\alpha - 3\sin\alpha$

Simplify: a $2 - 2\sin^2\theta$ b $\cos^2\theta\sin\theta + \sin^3\theta$

GOAL: Write in terms of 1 trig function and one term

Things to try: Factor; Substitution using identities, etc

These are typically "Show that" problems

Expand and simplify: $(\cos\theta - \sin\theta)^2$

Use an appropriate 'double angle formula' to simplify:

a $3\sin\theta\cos\theta$ **b** $4\cos^2 2B - 2$

Factorise: **a** $\cos^2\alpha - \sin^2\alpha$ **b** $\tan^2\theta - 3\tan\theta + 2$

Simplify: **a** $\dfrac{2 - 2\cos^2\theta}{1 + \cos\theta}$ **b** $\dfrac{\cos\theta - \sin\theta}{\cos^2\theta - \sin^2\theta}$

Simplify the following expressions

(a) $\cos\theta + \tan\theta \sin\theta$ (b) $\dfrac{\cos\theta}{1+\sin\theta} - \dfrac{1-\sin\theta}{\cos\theta}$

Prove that $\dfrac{\sin 2\phi + \sin\phi}{\cos 2\phi + \cos\phi + 1} = \tan\phi$

Prove that $\sin 2\alpha \tan\alpha + \cos 2\alpha = 1$

After factoring, we can solve the equations

Solve for $0 \leqslant x \leqslant 2\pi$ giving your answers as **exact** values:

$2\sin^2 x + \sin x = 0$

$2\cos^2 x + \cos x + 1 = 0$

Solve for $0 \leqslant x \leqslant 2\pi$ giving your answers as **exact** values:

$\cos 2x + 3\cos x = 1$

Find all values of x, such that $\sin 2x = \cos x$, where $0 \leq x \leq 2\pi$.

Homework

Chapter 13.3

13F: 1-7