Patterns and Sequences

Skills check

1 Solve each equation.

a $3x - 5 = 5x + 7$

b $p(2 - p) = -15$

c $2^n + 9 = 41$

2 Solve for k.

a $6m + 8k = 30$

b $2pk - 5 = 3$

3 If $T = 2x(x + 3y)$, then find the value of T when

a $x = 3$ and $y = 5$

b $x = 4.7$ and $y = -2$

4 Using the formula $m = 2^x - y^3$, find the value of m if

a $x = 5$ and $y = 3$

b $x = 3$ and $y = -2$

c $x = -5$ and $y = \frac{1}{2}$

Saving Money

Joel decides to start saving money. He saves $20 the first week, $25 the second week, and $30 the third week, and so on.

1. Complete the table below to show how much Joel saves each week, and how much he has saved in total, for the first 8 weeks.

Week Number	Weekly Savings	Total Savings
1	20	20
2	25	45
3	30	75
4		
5		
6		
7		
8		

2. How much will Joel save in the 10th week?

3. How much will Joel save in the 17th week?

4. How much money will Joel save in total the first year?

5. How long will it take for him to save a total of at least $1,000?

6. Try to write a formula for the amount of money Joel saves each week. Let ***M*** represent the amount of money he saves each week, and let ***n*** represent the week number.

$M = 15 + 5n$ $\quad 20 + 5(n-1)$

7. Try to write a formula for the **Total** amount of money Joel has saved. Let ***T*** represent his total savings, and let n represent the number of weeks.

$T = 2.5n^2 + 17.5n$

$T_n = M_n + T_{n-1}$

$T = [15 + 5n] + [15 + 5(n+1)] + [15 + 5(n+2)] + \dots$

What is a sequence???

A sequence . . .

- Is a list of terms (a term is the individual number or element)
- Can be specified by a rule/equation
- Can be infinite or finite
- Is a function whose input is consecutive integers and the output are the terms

The following are sequences:

A: 2, 4, 6, 8, 10, . . .

B: 1, 4, 9, 16, 25

C: -5, 10, -15, 20, -25, . . .

D: 3, 9, 15, 21 . . .

E: $\frac{2}{3}, \frac{4}{4}, \frac{6}{5}, \frac{8}{6}$

- Which of the sequences above are finite?
- Which are infinite?

Write the first six terms of the following sequences. Start with n = 1.

You are given the general term or rule for finding the value of the terms of the sequence.

1. $a_n = n^2 + 1$ 2, 5, 10, 17, 26, 37

2. $a_n = (-2)^{n-1}$ 1, -2, 4, -8, 16, -32

3. $f(n) = 3n + 5$

   8, 11, 14, 17, 20, 23

We have two ways to write the general term of a sequence:

1. Recursive Formulas - value of a term depends on the value of the previous term

2. General Formulas - value of a term depends on which term you are finding

- You can use the notation u_n *to denote the* n^{th} ***term*** *of a sequence, where* ***n*** *is a positive integer.*

So for 8, 11, 14, 17, … you could say
$u_1 = 8, \quad u_2 = 11, \quad u_3 = 14$, and so on.

- A **recursive formula** is where the value of a term depends on the value of the previous term.
- Notice that the each term is three greater than the value of the ***previous*** term.

8, 11, 14, 17, 20, 23, 26

- So we could write it in a recursive formula as:

$$\underline{u_1 = 8} \text{ and } u_{n+1} = u_n + 3$$

term you want to find

previous term

Write a recursive formula for the nth term of each sequence.

a 9, 15, 21, 27, …

$u_1 = 9$ and $u_{n+1} = u_n + 6$

b 2, 6, 18, 54, …

$u_1 = 2$ and $u_{n+1} = 3u_n$

Sometimes, it is much more useful to know the general formula for the n^{th} term so that you do not have to rely on knowing the value of the previous term.

For the sequence 1, 4, 9, 16, 25, ... What pattern do you notice?

What is the **general formula** in terms of n? $u_n = n^2$

What about the sequence 5, 10, 15, 20, 25, ... ? $u_n = 5n$

Go back to the sequences from before.
Describe the pattern and write a general rule for the n^{th} term

A: 2, 4, 6, 8, 10, . . . $a_n = 2n$

B: 1, 4, 9, 16, 25 $a_n = n^2$

C: -5, 10, -15, 20, -25, . . . $a_n = (-1)^n \cdot 5n$

D: 3, 9, 15, 21 . . . $a_n = 6n - 3$

E: $\frac{2}{3}, \frac{4}{4}, \frac{6}{5}, \frac{8}{6}$ $a_n = \frac{2n}{n+2}$

Write a general formula for the nth term of each sequence.

a 4, 8, 12, 16, …

b $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

a) $u_n = 4n$

b) $u_n = \frac{1}{3n}$

$\cancel{u_n = \frac{1}{n+3}}$

$\cancel{u_1 = \frac{1}{1+3} = \frac{1}{4}}$

Homework:
Chapter 6.1 6A: 1-6

Need extra practice?

Haese and Harris

Ch 2 A: 1-4

Ch 2 B: 1-3